This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

GODFREY LOWELL CABOT SCIENCE LIBRARY
of the Harvard College Library

This book is
FRAGILE
and circulates only with permission.
Please handle with care
and consult a staff member
before photocopying.

Thanks for your help in preserving
Harvard's library collections.

REPORT

ON THE EFFECT OF THE

CHICAGO DRAINAGE CANAL

ON THE LEVELS OF THE

GREAT LAKES

PRINTED BY ORDER OF PARLIAMENT

OTTAWA
PRINTED BY S. E. DAWSON, PRINTER TO THE QUEEN'S MOST EXCELLENT MAJESTY
1896

REPORT

ON THE EFFECT OF THE

CHICAGO DRAINAGE CANAL

ON THE LEVELS OF TH

GREAT LAKES

By
J. L. P. O'Hanly.

PRINTED BY ORDER OF PARLIAMENT

OTTAWA
PRINTED BY S. E. DAWSON, PRINTER TO THE QUEEN'S MOST EXCELLENT MAJESTY
1896

H80
12
~~E.5738~~
Eng 1048.96
SEP 9 1896
CAMBRIDGE, MASS.

Hon J. G. Blaine

REPORT

[82]

AD INTERIM REPORT ON THE EFFECT OF THE CHICAGO DRAINAGE CANAL ON THE LEVELS OF THE GREAT LAKES.

HON. JOHN COSTIGAN,
Minister of Marine and Fisheries,
Ottawa.

SIR,—I beg leave to report as follows:—

NARRATIVE.

When you did me the honour of communicating your intention of instituting an official inquiry into the probable effects of the Chicago Drainage Channel on the levels of the Great Lakes and their connecting rivers, you were good enough to ask my opinion as to the cost of the undertaking.

I informed you that the cost of such an investigation as you contemplated depended largely, if not wholly, on the scope of the inquiry. Were it, in character, original, the process would be slow, and necessarily involve considerable outlay, as there are no Canadian or English data on which to draw. In illustration, I instanced the costly and tedious survey of the Great Lakes by the Government of the United States. This survey was commenced in 1841, and completed in 1882, with some partial resurveys since. It has cost over three million dollars. The survey comprised the River St. Lawrence from the international boundary at St. Regis westward, Lake Ontario, the Niagara River, Lake Erie, the Detroit River, the St. Clair Flats, Lake and River St. Clair, Lakes Huron and Michigan, St. Mary's River and Lake Superior, not including Georgian Bay.

If, on the other hand, it were considered advisable to utilize the operations and researches of the American Government in the survey of the Great Lakes and kindred inquiries, I said that I thought it highly probable that the necessary data are available, and only require examination, collation, selection, and digestion; and that I considered the proposed investigation would require very little, if any, field operations.

I respectfully submitted that, in my humble opinion, the best, the most feasible, the most expeditious, the most economical policy, was to utilize the American data, which I believe to be thorougly reliable. In this opinion, you were good enough after due and careful deliberation to acquiesce; and to direct that the inquiry be conducted on these lines.

Accordingly, on August 19, 1895, I began operations. I examined the engineering and kindred records of the Deparment of Marine and Fisheries, kindly placed at my disposal by the officer in charge of that important branch of the public service. My researches here embraced: (1) An examination of the charts of the Great Lakes and connecting rivers, published at Washington under direction of the Chief of Engineers, United States Army, with memoranda. (2) A perusal of such of the annual reports of the United States Coast and Geodetic Survey as came within my purview, with notes.

Having exhausted all the known and available sources of information in this field, I transferred my operations to a new and more fertile one—the Library of Parliament. I examined all the annual reports of the Chief of Engineers of the United States Army contained in that institution, numbering over one hundred large volumes, and taking copious notes, as well as many congressional and other public documents of the United States Government, with elaborate memoranda.

It will readily be appreciated how slow and tedious is this process. It is like the proverbial "looking for a needle in a bundle of straw." After having perused and examined many voluminous documents, books and records, it often turns out they contain nothing to the purpose. One is also circumscribed by the rules of a public institution, and must conform to its hours and regulations. Indeed, one always labours under very great disadvantages under such circumstances; and the result seldom seems commensurate with the time occupied. The officers of the library, from the librarian down, were most kind, courteous and obliging, and evinced a cordial disposition to render every assistance in their power. Without making invidious distinctions, I am especially indebted to Mr. Smith for much useful information and other valuable services.

My search in the library justified and confirmed my first impression. I found that the records of the Corps of Engineers of the United States Army contained all or nearly all the data, as well as the only extant data, for the determination of this unique and important problem. While thus engaged, I learned, through the press, for the first time, that the United States Government had appointed a Board of Engineers to report on the very subject with which I was dealing, and that the Board had concluded its labours and submitted its report to the Chief of Engineers. This, I think, was towards the end of September.

I was hoping that this report would forthwith be published, and anxiously awaited its receipt in the library. Weary of waiting, I applied to Hon. J. B. Riley, American Consul General, Ottawa, to use his good offices in procuring me a copy from Washington. Mr. Riley very kindly wrote to the Chief of Engineers, General William P. Craighill, Washington, D.C., to send a copy, if published. To this, General Craighill replied, that the report remained yet unpublished, but that, with the permission of the Chief of Engineers, it had been printed in the New York *Engineering News*, of October 3, 1895. Through a local news agency, I secured a copy of that issue. I wish here to signify and acknowledge my deep obligations to Mr. Riley for this and many other favours in connection with this service.

A copy of the report is herewith annexed, marked Appendix A.

Perhaps there was no man living more conversant with the scientific history—the Physics and Hydraulics of the Great Lakes—than the President of the Board, the late General Poe. He was connected with the Lake Survey, almost from its inception, continuously to the day of his lamented death. Since 1882 he was chief of the Northern Division. It has been under his efficient supervision that all the great improvements to the navigation of the Great Lakes from Duluth to Buffalo, now drawing to a successful close, took place.

The Board puts this question: "What is the outflow of the lower lakes?" To which they make the subjoined reply:

"In November, 1891, the Chief of Engineers, U.S.A., * * * ordered a set of observations made to determine the amount of water flowing down the Niagara River. * * *

"The results of these measurements were somewhat unexpected, and they were repeated in May, 1892. The second set corroborated the first, and the whole formed a subject of a report to the Chief of Engineers, which appeared in his annual report of 1893. * * This publication was the first ever made in which, as a matter of careful measurements, a relation between the level of the lakes and their outflow, or discharge, had been established and given to the public."

Continues the report: "Will the loss be six inches, or will it be three inches? This is an important question, and we have only the Niagara River discharge observations from which to answer it. These cover a range of about 1·8 feet. There were scattering observations outside these limits, but the mass of results were secured between gauge readings, mean lake level, the highest, and 1·85 feet. The 'smooth curve' * * enables us to note the fall of 0·53 feet on the gauge per 10,000 cubic feet per second for the first foot of fall, and 0·44 feet for the whole.

"* * * This lower portion of the gauge should be studied and additional observations made, and the Board is a unit in suggesting the importance of a series of gaugings of the St. Clair River at the present time for this purpose, and to furnish

additional knowledge of the relation between gauge readings and discharge. The subject is of such general bearing upon the navigation of the lakes that it demands careful treatment and full data. * * * *

"The abstraction of 10,000 cubic feet of water per second from Lake Michigan will lower the levels of all the lakes of the system except Lake Superior, and reduce the navigable capacities of all harbours and shallows throughout the system to an extent that may be determined, if at all, by actual measurements only."

The Board of Engineers are unanimously of opinion that the Niagara River discharge measurements of 1891 and 1892 are the only reliable and trustworthy gaugings hitherto made of the outflow of the Great Lakes, and they unhesitatingly concur that it is wholly inadequate to determine the effect of the Chicago Drainage Channel on the levels of the lakes and their connecting waterways. They recommend that these be supplemented with a series of discharge measurements of the St. Clair River, as indispensable to determine this unique and intricate hydraulic problem.

I have since learnt, through the courtesy of the Chief of Engineers, United States Army, and politeness of Col. A. Mackenzie, Corps of Engineers, that nothing has yet been done towards carrying out the recommendation of the Board of Engineers. Nor is any intimation vouched of what is intended.

A copy of Col. Mackenzie's letter will be found in Appendix B, accompanying this report.

It is obvious that if the data be considered insufficient for the uses of the American Government, it is so likewise for those of the Canadian. It therefore remains for the department to determine what course to pursue in prosecuting the inquiry to a successful issue. May I be permitted to interject an opinion, I would respectfully suggest that a gauging party be organized, under a competent head, with the opening of spring, to make discharge measurements of the St. Clair River at some suitable point between Sarnia and Mooretown, to be continued until the close of the season, in order to make measurements at all stages of the river during the season of navigation. This operation might either be carried on independently, or in conjunction with the United States party.

LEVELS OF THE GREAT LAKES.

The cheapest and most expeditious way of making a pond is to follow the example of the beaver—dam a living stream. Given certain conditions—slope or declivity of stream and average cross section of valley—and the volume of impounded water will depend on the height of the dam. But this height has its limits. The elevation of the crest of the dam must not exceed the elevation of the summit of the lowest depression in the drainage basin above the site of the dam. For the water on exceeding this height would be diverted to a new route, flow in a new bed or channel, at least during the lifetime of the dam, or while its crest remained at its present elevation. A case may be readily conceived in which the water would flow diametrically opposite to its original or normal course.

The volume of water discharged over the dam depends on the length of the weir, the elevation of its abutments above the crest, and the depth of water on the crest. In a normal condition of flow, with the pond in a state of equilibrium, that is to say, the water quiescent, its surface neither rising nor falling, the discharge over the weir would equal the entire inflow from the drainage area above the dam, *minus* evaporation. This normal condition of flow or state of equilibrium is generally designated mean level of the pond.

Evaporation is the source of rain, and precipitation of supplies of water. The unabsorbed percentage of the rain falling on the drainage basin, above the dam, constitutes its influx or supply. The precipitation being an uncertain and variable *quantum*, the pond is rarely or never in a state of equilibrium. Its fluctuations aptly resemble the oscillations of a pendulum—ever tending towards the vertical, but never, except for an infinitesimal instant, remaining there. In a wet season the precipitation is excessive. The supply being augmented, the level of the pond rises, and may sometimes overflow

its banks. It is a well-known law of hydraulics that the discharge varies with the head of pressure. If at a certain head there is a fixed or determinate discharge over a weir, drowned or not, then if you wish to increase the discharge you must increase the head. On the contrary, in a dry season, the supply is stinted, diminished, and the level of the pond, in obedience to the above law, falls. Any deterioration of the dam, either by erosion and lowering of its crest or by filtration through its crevices, depresses the level of the pond. There is still another way by which the surface of the water in the pond may be lowered to any conceivable extent, viz.: by digging an open ditch or tunnel, diverting all or any assignable part of its waters in an artificial channel, the quantity diverted depending on the slope and dimensions of the new conduit.

Suppose that our weir is 200 feet long, with a depth of 6 inches of water immediately behind the crest of the dam, designed to discharge, in a mean or normal stage of supply, 100 cubic feet per second. The phenomenon of discharge consists in this: That every particle of water entering at the inlet forces out an equal particle, bulk for bulk or weight for weight, at the outlet. The water itself is inert. It derives its force from the acceleration of gravity imparted to the particle at the inlet. It acts as a lever to push forward the inert mass between it and the outlet, indifferently whether the intervening mass is in a solid or fluid state. It may aptly be compared to a solid, say, a sheet of metal 200 feet wide, 6 inches deep, and continuously extending from inlet to outlet, floating on the quiescent fluid beneath. Any compressive force applied at the upper end will push out the sheet at the outlet end. Nor would it alter the principle of action if the body of water on which this sheet floats were converted into a solid. For all the liquid below a horizontal plane tangent to the crescent of the dam is quiescent or motionless, unless disturbed or agitated by extraneous causes, as winds, earthquakes and other seismic vibrations, the screw or paddles of a steamboat, &c., &c. This motion is called a motion of translation, and is such that any one particle has an equal and parallel motion with every other particle of the body. It is characteristic of the motion of a solid in a rectilineal direction.

Now every lake, great and small, is in the condition of our pond. The barrier or obstruction at its outlet is a submerged dam, resembling our weir, with banks for abutments, on whose elevation, slope and dimensions chiefly depends the permanent elevation of the surface of the water, or "mean lake level," as it is called. Had there been no obstruction at the outlet, there would have been no lake. The laws governing these phenomena are precisely the same, whether the dam was erected by human hands, fashioned out in nature's laboratory, or contrived by the ingenious and persevering beaver.

HYDRAULICS.

The science of hydraulics is pre-eminently inductive or experimental. It is very unsafe to predicate a formula on any of the observed phenomena of flowing water until verified by experiment. This arises from no analytic defect in the demonstration, but because the conditions of any two cases are seldom or never exactly alike. We can compute theoretically, within reasonable limits of accuracy, the discharge over a weir, or through a regular channel, as a canal, with given slope and dimensions. But when it comes to gauging a large river like the St. Lawrence or any of the connecting links in the great system of our inland seas, hydraulic formulæ lose their magic, and theory and speculation have to step aside and make way for experiment. Even with modern improvements in methods and instruments, discharge measurements are not only a delicate operation, but are still hemmed round with much uncertainty.

Much as are discharge computations surrounded, if not involved in mystery, they are simplicity itself compared to any approximate, much less exact, determination in advance of the effect of the Chicago Drainage Channel on the levels of the Great Lakes. It is obvious to the plainest undertanding that the surface of the lakes must, to some extent, be lowered by the opening of the new outlet, or more properly, perhaps, ancient outlet, resuscitated, at Chicago. The project in theory contemplates an ultimate draught on the reservoir of 10,000 cubic feet per second at the lowest stage of Lake Michigan.

With the known fluctuations of the lake, this will often exceed 15,000 cubic feet per second. To believe that this will not prejudicially affect the levels of all the lakes, except Lake Superior, as well as all their connecting waterways and outlet, would be equivalent to believing an absurdity. The commonest understanding has no difficulty in believing and appreciating this, the greatest intellect cannot soar much higher, come much nearer to a solution.

The problem is complex and intricate as unique. No hydraulic formula is applicable to it, for the simple reason that no such contingency has hitherto arisen, or is likely to ever again arise. It is so beset with extraneous, disturbing elements, impossible of elimination, as to be practically indeterminate by any known hydraulic formulæ. Among these are storms, changes of barometric pressure, changes in rainfall and in the percentage of the precipitation absorbed, &c., &c., &c. It is recorded that in the storm on Lake Erie of October 14, 1893, there was a difference of elevation of nearly twelve feet between the western and eastern ends of the lake. That is to say, at Toledo the water fell 6 feet 8 inches below the then normal level of the lake, while at Buffalo the water rose 5 feet 3 inches, or a total difference of elevation of 11 feet 11 inches, between these two places. (See accompanying Plate No. 4, copied from the annual report of the Chief of Engineers, United States Army, for 1894.)

An idea of the complexity of the problem may be gleaned from the following assumption: Suppose that nature, in her inscrutable ways, instead of inspiring her pigmy offspring with the conception of the construction of an artificial channel from Lake Michigan to the Mississippi River, capable of discharging, at the lowest stage of lake level, not less than 10,000 cubic feet of water per second, augumenting with the increased head of pressure to a probable discharge at mean lake level of 15,000 cubic feet, and a maximum discharge at highest lake level of 17,000 cubic feet; had, in her own laboratory, fashioned and chiselled out a subterranean passage of equal capacity with the proposed Chicago drainage ditch, fitting the outlet with automatic valves and sluices to regulate the outflow in such a manner that like discharges would take place under similar heads of pressure. With present knowledge of the physics and hydraulics of the Great Lakes, it would, perchance, take a century of minute, elaborate scientific research to determine whether one gallon of Lake Michigan's waters had escaped by other than the normal or natural processes—discharge through the Straits of MacKinac into Lake Huron, and absorption into the atmosphere through evaporation. It is not improbable that at the close of the century's investigations, science would retire baffled, discomfited in the contest with nature, the mystery still unsolved, that the loss would be still masked, eclipsed, concealed from human ken, unless disclosed by accident, while the fact remained that Lake Michigan was bled during every second of that century to the tune of 12,500 cubic feet, with the doctors still diagnosing the patient. The discharge would aggregate to the enormous quantity of nearly forty trillions (39,446,161-250,000) cubic feet, a quantity too vast to be comprehensible to the most trained intellect. Some tangible idea of its vastness may be presented to the mind by clothing it in the concrete form of a mass of water occupying a hollow prism or parallelepipedon, 283 miles long, 50 miles wide and 100 feet deep.

The Board of Engineers, in their report on this phase of the inquiry, says: "The abstraction of 10,000 cubic of water per second from Lake Michigan will lower the levels of all the lakes of the system, except Lake Superior, and reduce the navigable capacities of all harbours and shallows throughout the system to an *extent that may be determined, if at all, by actual measurement only.*" (The italics are mine.)

Again they say, "The water levels of the great lakes are very delicate."

Mark the phraseology used by the Board. The effect on the levels of the Great Lakes of the abstraction of 10,000 cubic feet per second from Lake Michigan, may be determined, if such determination be at all possible, "by actual measurement only." Manifestly the Board entertain grave doubts of its possibility under any circumstances. But if at all possible, it is only by pursuing the *modus* they recommend.

DESCRIPTIVE.

The brief, yet precise, description of the contemplated works given by the Board of Engineers admirably answers the needs and purposes of this report.

"The Main Drainage Channel of the Sanitary District of Chicago is now under contract from its confluence with the south branch of the Chicago River, at Roby Street, in the City of Chicago, to its southern terminus, in Will County, Ill. At the southern end of the channel the controlling works will be located. Beyond these works, the construction contemplated by the district will be the work necessary for conducting the flow of the Desplaines River, down the declivity to and through the City of Joliet, and making such change in the Illinois and Michigan Canal as the new conditions developed will make necessary.

"The first work put under contract extended south-westerly from the Willow Springs road, and these sections were numbered consecutively Nos. 1 to 14. Average length of sections one mile. Easterly from Willow Springs road the sections are lettered from A to O, omitting J. The lettered sections are, except for a short distance near Summit, entirely in glacial drift, defined in the specifications thus: 'Glacial drift shall comprise the top soil, earth, muck, sand, gravel, clay, hard pan, boulders, fragmentary rock displaced from its original bed, and any other material that overlies the bedrock.'

"The sections from 1 to 14 were put under contract in July, 1892; from A to F were put under contract late in 1892 and early in 1893; and G to M, inclusive, were contracted for in December, 1893. Sections N and O were put under contract May 2, and section 15, August 27, 1894. Earth was first broken on "Shovel Day," September 3, 1892, on the rock cut below Lemont.

"The Desplaines Valley is traversed by the river from which it takes its name, a stream of wide fluctuations, with no constant and reliable fountain supply. During some seasons its whole discharge would pass through a 6-inch pipe, and at others its volume reaches 800,000 cubic feet per minute. Then it rolls majestically along, flooding the whole valley. Such being the situation, control of this stream was a condition precedent to the successful prosecution of the work upon the main channel. This control has been secured by the outlay of nearly $1,000,000 in constructing what is known as the River Diversion channel.

"About 13 miles of new river channel had to be excavated with the location of the Main Drainage Channel, and about 19 miles of levee built to divorce the waters of the Desplaines watershed from the channel which is to receive the waters of Lake Michigan and pass them on to the Mississippi River *via* the lower Desplaines and the Illinois rivers. The width of the River Diversion Channel on the bottom is 200 feet, side slopes 1½ to 1, grade generally 0·12 ft. per 1,000 ft.

"At the head of this river diversion it was necessary to provide a safety valve in the form of a spillway to allow surplus water to flow toward Chicago, because arrangements have not as yet been perfected for carrying the entire flood waters of the Desplaines through Joliet.

"This spillway is a concrete dam capped with cut stone, and its wings faced with stone masonry; it is 397 ft. long, and its crest is 16·25 ft. above Chicago datum. (This datum is referred to the low water of Lake Michigan of 1847, and is 579·61 ft. above sea level at Sandy Hook). No water flows over this spillway until the volume passing the water gauge above it reaches 300,000 cubic feet per minute.

"The cross section of the earth sections from A to E, inclusive, is 202 ft. on the bottom, with side slopes of 2 to 1. This section extends for about 500 ft. into the west end of F, and then reduces to 110 ft. on the bottom, preserving the same side slopes. The explanation for this change of cross section is as follows: Throughout the rock sections, and those sections in which there is a preponderance of hard material, or where rock may appear, the section adopted is designed according to law for a flow of 600,000 cubic feet of water per minute, which means provision for a population of 3,000,000 people. The narrow channel provides for a flow 300,000 cubic feet per minute, or for about the present population of Chicago. The enlargement of the narrow channel can be made by the easier methods of excavation, such as dredging, whenever the needs of

the city require it. The grade throughout the lettered sections is 1 ft. in 40,000 (·025 ft. per 1,000 ft.), and the bottom of the channel at Robey Street is 24,448 ft. below datum. The numbered sections, from No. 1 to No. 6 inclusive, are underlaid with solid rock. The width of the bottom, in rock, is 160 ft., and walls of masonry laid in cement will be built upon the rock surface to a height of 5 feet above datum. Sections 7 to 14, inclusive, are in solid rock; width at bottom, 160 ft.; sides vertical, prism taken out in three steps with offsets of 6 inches on each side for each cut, making top width of 162 ft., grade in rock, 1 ft. in 20,000 (·05 ft. per 1,000).

"Section No. 15 is also in rock, and its cross section is enlarged at its south end, so as to form a 'windage basin' in which large vessels may be turned around. The controlling works are located on this section. These works will consist of gates or movable dams, by which the flow of water from the main channel into the tail race, which is to deliver the outflow into the Desplaines River, can be controlled.

"This river below Lockport follows the trough of the valley down a steep declivity to the canal basin in Joliet. The fluctuations in Lake Michigan, by varying slope of water surface, will be felt at the controlling works, and provision must be made to meet these fluctuations within a range of five ft. above datum and 8 ft. below, or an extreme oscillation of 13 ft. The fall from datum at the controlling works to the level of the upper basin will be about 42 ft. in a distance of about 4⅓ miles. As the plans for controlling works have not been finally adopted by the Board of Trustees, they cannot now be discussed.

"The total amount of excavation involved in the construction of the main channel is 26,077,765 cubic yds. of glacial drift, and 12,071,668 cubic yds. of solid rock, or an aggregate of 38,149,433 cubic yds., to which must be added the material excavated from the river diversion: Glacial drift, 1,564,403 cubic yds.; solid rock, 258,926 cubic yds.; total river diversion, 1,823,329 cubic yds.; grand total, main channel and river diversion, 39,972,762 cubic yds. All of this work is now under contract, and in addition thereto 384,958 cubic yds. of retaining wall."

The subjoined supplemental description is extracted from the paper of Lyman E. Cooley, Esq., C.E., M. Am. Soc. C. E., and a member of the Board of Trustees of the Sanitary District of Chicago:—

EFFECT OF THE CHICAGO DRAINAGE CANAL ON LAKE LEVELS.

"The work as now laid out provides for a main channel, which begins at a junction with the Chicago River, or rather the south branch thereof, in the south-west quarter of the city, at a point 5·8 miles from Lake Michigan, by the course of the river, and extends to the vicinity of Lockport, a distance of 28·05 miles from the point of beginning, where the water is to be discharged in to the Desplaines River, and such work done along the bed thereof, for a distance of 7·1 miles, as is necessary to conduct the overflow safely through the city of Joliet.

"For 7·8 miles out from Chicago the channel is being constructed with a present capacity of 5,000 cubic ft. per second, the future enlargement being simply a matter of dredging through comparatively easy material. The 20·25 miles in the Desplaines Valley is through glacial drift of the most difficult character and through rock, and this part of the work is being carried out on the ultimate basis of capacity. The standard dimensions as adopted are, for 14·9 miles through the rock cut, 160 ft. wide at bottom and 162 ft. at the top, with a declivity of one foot in 20,000 ft.; and in the earth and drift for a distance of 13·15 miles, 202 ft. wide at bottom and 290 ft. at the water line when the channel is carrying 22 ft. of water, with a declivity of 1 ft. in 40,000 ft., excepting, however, the 7·8 miles at the Chicago end, previously mentioned, which are being constructed with a width 92 ft. narrower than the standard earth section.

"The bottom of the channel, at its junction with the Chicago River, is actually 24·45 below datum, and at the Lockport end 30 ft., the total theoretical declivity being 5·65 ft. The capacity is figured on a depth of 22 ft. on a conservative basis, so as to make sure of meeting any requirement of the inspectors, who are to be independent agents of the state. It will be noticed that an allowance of 2·45 ft. is made in the

grade at the Chicago end in order surely to meet any solution that may be demanded through a connection with Lake Michigan, in order to feed this channel to its full capacity. These additional works have not been provided for, nor have plans for the same been matured. Should these works be carried out on a liberal basis, the depth in the main channel will be increased 2 ft. at low water.

"It is proposed to open these works on the minimum capacity provided by law (assumed at not less than 5,000 cubic ft. per second, but actually 20,000 cubic ft. per minute for each 100,000 of population), and it is presumed that considerable work will be required in the Chicago River to pass the minimum volume without injury to navigation. Extensive works of a radical nature will be necessary to provide the ultimate volume, viz. : 10,000 cubic ft. per second, and several years will be required for their development.

"The volume flowing in this channel will be regulated by controlling works at the lower end at Lockport, and by these means the discharge may be fixed and controlled at any amount or entirely stopped at pleasure.

"Aside from its sanitary utility, the channel is to be regarded as the most costly part of the waterway from Lake Michigan to the Mississippi, and as a useful extension of the harbour of Chicago for vessels of deep draft, and these objects were centemplated as important incidents of the work, and were fully expressed as the policy of the state when the law was passed."

HYDRAULIC FORMULÆ.—DISCHARGE COMPUTATIONS.

As the mode of connection of the Chicago Drainage Channel with Lake Michigan is yet undetermined, all that is known with certainty of the contemplated draught on the lake is that there will be a minimum depth of 22 ft. of water in the channel at the junction of the drainage channel with the south branch of the Chicago River, at Robey Street, which is 5·8 miles by river from the lake. The elevation of the bed or bottom of the channel at this point is 24·45 feet below Chicago datum. Hence the elevation of the water surface in the channel will be 2·448 feet below Chicago datum. This is equivalent to—4·7 + 2·448 = —7·148 feet below the plane of reference of the water level curves of Lake Michigan, as adopted by the United States Lake Survey.

The monthly mean level of Lake Michigan at Milwaukee, Wis., for the month of November, 1895, was—5·642 feet below the said plane of reference. This has been the lowest stage of the water in Lake Michigan, of which there is any record, up to date. Whether it has fallen still lower since, I am not aware. This would be equivalent to an elevation of the water surface of Lake Michigan of 1·5 feet above the water surface of the drainage channel at Robey Street junction with its minimum depth of 22 feet.

Assuming that for every oscillation of Lake Michigan there will be an equal and corresponding fluctuation of the drainage channel at Robey Street junction. Then, 2ndly, when the water of Lake Michigan will be one foot above the elevation of November, 1895, which nearly coincides with the low water of March, 1869, there will be a depth of 23 feet in the drainage channel at the Robey Street junction.

3rdly. When the water of Lake Michigan attains an elevation of two feet above that of November, 1895, there will be a depth of 24 feet in the drainage channel at Robey Street junction. This elevation nearly coincides with the low water of January, 1870, and February, 1888.

4thly. When the water of Lake Michigan attains an elevation of three feet above that of November, 1895, there will be a depth of 25 feet in the drainage channel at Robey street junction. This elevation nearly coincides with the low water of March, 1863, of January, 1884, of January, 1885, and of January, 1887 ; and with the high water of August, 1865, of June, 1867, of August, 1873, of September, 1875, and of August, 1880.

5thly. When the water of Lake Michigan attains an elevation of four feet above that of November, 1895, there will be a depth of 26 feet in the drainage channel at Robey street junction. This elevation nearly coincides with the high water of July, 1860, and July, 1862.

6thly. When the water of Lake Michigan attains an elevation of five feet above that of November, 1895, there will be a depth of 27 feet in the drainage channel at Robey Street junction. The highest water of Lake Michigan since 1859, was in July of that year. It was only—0·25 feet below the stage of five feet above the low water of November, 1895. This stage is equivalent to 0·642 feet below the plane of reference, or supposed high water of 1838.

HYDRAULIC FORMULÆ.

DISCHARGE COMPUTATIONS.

1ST.—ELEVATION OF LAKE SURFACE COINCIDENT WITH LOW WATER OF NOVEMBER, 1895.

1ST.—CHANNEL IN EARTH.

CROSS-SECTION.

Width at bottom ..... 202 feet.
Depth of water ..... 22 "
Side slopes ..... 2 horizontal to 1 vertical.
Width at surface of water ..... $(101 + 44)2 = 290$ feet.
Sectional area A ..... $\frac{202 + 290}{2} \times 22 = 5{,}412$ square feet.
Wetted perimeter ..... $(101 + \sqrt{22^2 + 44^2}) \times 2 = (101 + 49) \times 2 = 300$ feet.
Hydraulic mean radius ..... $\frac{5412}{300} = 18$ feet.
Slope of channel ..... 1 in 40,000, or ·000025 feet per foot linear.

1.—Kutter's Formula.

Assumed roughness of channel, $n = \cdot 0275$, a mean between ·025 and ·03.

Mean velocity—

$$v = \left\{ \frac{41{\cdot}6 + \frac{1{\cdot}811}{n} + \frac{{\cdot}00281}{i}}{1 + \left(41{\cdot}6 + \frac{{\cdot}00281}{i}\right)\frac{n}{\sqrt{m}}} \right\} \sqrt{mi}$$

$$= \left\{ \frac{41{\cdot}6 + \frac{1{\cdot}811}{{\cdot}0275} + \frac{{\cdot}00281}{{\cdot}000025}}{1 + \left(41{\cdot}6 + \frac{{\cdot}00281}{{\cdot}000025}\right)\frac{{\cdot}0275}{\sqrt{18}}} \right\} \sqrt{18 \times {\cdot}000025}$$

$$= \left\{ \frac{41{\cdot}6 \quad 65{\cdot}85 \quad 112{\cdot}4}{1 \quad (41{\cdot}6 \times 112{\cdot}4) \times {\cdot}00648} \right\} \times {\cdot}0212$$

$$\frac{219{\cdot}85}{2} \times {\cdot}0212 = 2{\cdot}33 \text{ feet per second.}$$

2.—de Prony's Formula.

Mean velocity—

$$v = ({\cdot}0237 + 9966\, m\, i)^{\frac{1}{2}} - {\cdot}1542$$

$$= ({\cdot}0237 + 9966 \times 18 \times {\cdot}000025)^{\frac{1}{2}} - {\cdot}1542$$

$$= ({\cdot}0237 + 4{\cdot}485)^{\frac{1}{2}} - {\cdot}1542$$

$$= 2{\cdot}1233 - {\cdot}1542 = 1{\cdot}969 \text{ feet per second.}$$

3.—Dupuit's Formula.

Mean velocity—

$${\cdot}0000323\, v^2 = i\,(1 + {\cdot}225) = (1 + {\cdot}225 \times 18) \times {\cdot}000025 = {\cdot}00012625$$

$$v^2 = \frac{{\cdot}00012625}{{\cdot}0000323} = 3{\cdot}9\text{; and } v = \sqrt{3{\cdot}9} = 1.975 \text{ feet per second.}$$

4.—Chezy's Formula.

Mean velocity—

$v = c\sqrt{m\,i}$, making $c = 100$. Substitute

$v = 100\sqrt{18 \times \cdot00025}$

$= 100\ \sqrt{\cdot00045}$

$= 100 \times .0212$

$= 2\cdot12$ feet per second.

5.—Humphrey's and Abbot's Formula.

Mean velocity—

$$v = \left\{\left(225\,\frac{m}{2}\sqrt{i}\right)^{\frac{1}{2}} - \cdot0388\right\}^2$$

$$= \left\{\left(225 \times \frac{18}{2}\sqrt{\cdot000025}\right)^{\frac{1}{2}} - \cdot0388\right\}^2$$

$$= \left\{\left(225 \times 9 \times \cdot005\right)^{\frac{1}{2}} - \cdot0388\right\}^2$$

$= (1\cdot78 - \cdot0388)^2$

$= 1\cdot74)^2 = 3$ feet per second.

6.—Hagen's Formula.

Mean velocity—

$$v = 4\cdot39\sqrt{m}\sqrt[6]{i}$$

$$= 4\cdot39\sqrt{18}\sqrt[6]{\cdot000025}$$

$= 4\cdot39 \times 4\cdot22 \times \cdot171$

$= 3\cdot168$ feet per second.

7.—de Prony's with Weisbach's Coefficients.

Mean Velocity—

$$v = \left(\cdot00024 + 8675\,m\,i\right)^{\frac{1}{2}} - \cdot015$$

$$= \left(\cdot00024 + 8675 \times 18 \times \cdot000025\right)^{\frac{1}{2}} - \cdot015$$

$$= \left(\cdot00024 + 3\cdot90375\right)^{\frac{1}{2}} - \cdot015$$

$= \sqrt{3\cdot904} - \cdot015$

$= 1.97 - \cdot015 = 1\cdot955$ feet per second.

8.—Rankine's Formula.

Mean velocity—

$$v = 8\cdot025\sqrt{\frac{i\,m}{\cdot007565}} = \sqrt{8512\,i\,m} = 92\cdot26\ \sqrt{i\,m}$$

$= 92\cdot26\sqrt{\cdot000025 \times 18}$

$= 92\cdot26 \times \sqrt{\cdot00045}$

$= 92\cdot26 \times \cdot0212 = 1{,}956$ feet per second.

Mean velocities—

1. By Kutter's formula..... $v = 2\cdot32$ feet per second.
2. " de Prony's " ..... $v = 1\cdot969$ "
3. " Dupuit's " ..... $v = 1\cdot975$ "
4. " Chezy's " ..... $v = 2\cdot12$ "
5. " Humphreys' and Abbot's formula..... $v = 3\cdot00$ "
6. " Hagen's formula..... $v = 3\cdot168$ "
7. " de Prony's with Wiesbach's coefficients..... $v = 1\cdot955$ "
8. " Rankine's formula..... $v = 1\cdot956$ "

$8)\overline{18\cdot435}$ "

Grand mean velocity..... $v = 2\cdot304$ "

To find the probable errors by the method of least squares, assuming each of the formalas to have equal weights.

No. of Formula.	Velocity.	Difference from Mean.	Square of Residual Errors.
1.	2·32	− 0·016	0·003
2.	1·969	+ 0·335	0·1122
3.	1.975	+ 0·329	0·1082
4.	2·12	+ 0·184	0·0239
5.	3·00	− 0·696	0·4844
6.	3·168	− 0·864	0·7465
7.	1·955	+ 0·309	0·0955
8.	1·956	+ 0·308	0·0949
Mean =	2·304		$[n\,n_1] = 1·6659$

Probable error :

$$r = ·6745\sqrt{\frac{[n\,n_1]}{m-1}} = ·6745\sqrt{\frac{1·6659}{8-1}}$$
$$= ·6745\,\sqrt{0.2380} = ·6745 \times \pm 0·47$$
$$= \pm 0·317 \text{ feet.}$$

Probable error of arithmetical mean :

$$r(x) = \frac{·6745}{\sqrt{m}}\sqrt{\frac{[n\,n_1]}{m-1}} = \frac{·6745}{\sqrt{8}}\sqrt{\frac{1·6659}{8-1}}$$
$$= \frac{·6745}{2·8284}\sqrt{0·2380} = ·2385 \times \pm 0·47$$
$$= 0·112 \text{ feet.}$$

Discharge, $D = 5412 \times 2·3 = 12{,}448$ cubic feet per second.

2ND. CHANNEL IN ROCK.

CROSS SECTION.

Bottom width..... 160 feet.
Width at water surface..... 162 "
Mean width..... 161 "
Depth..... 22 "
Sectional area, $A = 161 \times 22 =$ .....3,542 square feet.
Wetted perimeter..... 204 feet.
Hydraulic mean radius, $m =$ ..... $\left(\frac{3542}{204} = 17·36 \text{ feet..}\right)$

Slope, $i =$ ..... $\left(\frac{1}{20{,}000} = ·00005 \text{ feet per linear foot.}\right)$

1.—Kutter's Formula.

Taking coefficient of roughness ·017, same as for rubble masonry.

Mean velocity –

$$v = \left\{\frac{41·6 + \frac{1·811}{n} + \frac{·00281}{i}}{1 + \left(41·6 + \frac{·00281}{i}\right)\frac{n}{\sqrt{m}}}\right\}\sqrt{m\,i}$$
$$= \left\{\frac{41·6 + \frac{1·811}{·017} + \frac{·00281}{·00005}}{1 + \left(41·6 + \frac{·00281}{·00005}\right)\frac{·017}{\sqrt{17·36}}}\right\}\sqrt{17·36 \times ·00005}$$
$$= \left\{\frac{41·6 + 106.5 + 56·2}{1 + (41·6 + 56) \times ·004}\right\} \times ·0298$$
$$= \frac{204·3 \times ·0298}{1 + ·3804}$$
$$= \frac{6·088}{1·38} = 4·4 \text{ feet per second.}$$

2.—de Prony's Formula.

Mean velocity—

$$v = \left(\cdot 0237 + 9966\ m\ i\right)^{\frac{1}{2}} - \cdot 1542$$

$$= \left(\cdot 0237 + 9966 \times 17 \cdot 36 \times \cdot 00005\right)^{\frac{1}{2}} - \cdot 1542$$

$$= \sqrt{8 \cdot 68} - \cdot 1542$$

$$= 2 \cdot 95 - \cdot 1542$$

$$= 2 \cdot 79 \text{ feet per second.}$$

3.—Dupuit's Formula.

Mean velocity—

$$\cdot 0000323\ v^2 = i(1 + \cdot 225m).$$

$$= (1 + \cdot 225 \times 17 \cdot 36) \times .00005$$

$$= (1 + 3 \cdot 906) \times \cdot 00005 = 4 \cdot 906 \times \cdot 00005$$

$$= \cdot 000249$$

$$\therefore \quad v^2 = \frac{\cdot 000249}{\cdot 0000323} = 7 \cdot 713\text{; and } v = 2 \cdot 78 \text{ feet per second.}$$

4.—Chezy's Formula.

Mean velocity—

$v = c\sqrt{m\,i}$. Calling $c = 100$. Then

$$v = 100\ \sqrt{17 \cdot 36 \times \cdot 00005}$$

$$= 100 \times \cdot 0295 = 2 \cdot 95 \text{ feet per second.}$$

5.—Humphreys and Abbot's Formula.

Mean velocity—

$$v = \left\{\left(225\ \frac{m}{2}\ \sqrt{i}\right)^{\frac{1}{4}} - \cdot 0388\right\}^2$$

$$= \left\{\left(225 \times \frac{17 \cdot 36}{2}\ \sqrt{\cdot 00005}\right)^{\frac{1}{4}} - \cdot 0388\right\}^2$$

$$= \left\{\left(19 \cdot 53 \times \cdot 007\right)^{\frac{1}{4}} - \cdot 0388\right\}^2$$

$$= \left(1 \cdot 9229 - \cdot 0388\right)^2$$

$$= 3 \cdot 55 \text{ feet per second.}$$

6.—Hagen's Formula.

Mean velocity—

$$v = 4 \cdot 39\sqrt{m} \cdot \sqrt[6]{i}$$

$$= 4 \cdot 39\ \sqrt{17 \cdot 36} \cdot \sqrt[6]{\cdot 00005}$$

$$4 \cdot 39 \times 4 \cdot 17 \times \cdot 191 = 3 \cdot 499 \text{ feet per second.}$$

7.—De Prony's Formula with Wisebach's coefficients.

Mean volocity—

$$v = \left(\cdot 00024 + 8675\ m\ i\right)^{\frac{1}{2}} - \cdot 015$$

$$= \left(\cdot 00024 + 8675 \times 17 \cdot 36 \times \cdot 00005\right)^{\frac{1}{2}} - \cdot 015$$

$$= \left(\cdot 00024 + 7 \cdot 53\right)^{\frac{1}{2}} - \cdot 015$$

$$= \sqrt{7 \cdot 53024} - \cdot 015$$

$$= 2 \cdot 745 - \cdot 015$$

$$= 2 \cdot 73 \text{ feet per second.}$$

8.—Rankine's Formula.

For first approximation, $f' = \cdot 007565$

Mean velocity—

$$v' = 8\cdot025 \sqrt{\frac{i\,m}{\cdot007565}} = \sqrt{8512\,i\,m} = 92\cdot26\sqrt{i\,m}$$

$$= 92\cdot26\ \sqrt{\cdot00005 \times 17\cdot36}$$

$$= 92.26\ \sqrt{\cdot000868}$$

$$= 92\cdot26 \times \cdot0295 = 2\cdot72 \text{ feet.}$$

For a second approximation.

$$f = \left(\cdot0074 + \frac{\cdot00023}{2\cdot72}\right) = \cdot0074 + \cdot0000848 = \cdot0074848$$

$$\tfrac{3}{2} - \frac{f}{\cdot01513} = \tfrac{3}{2} - \frac{\cdot0074848}{\cdot01513} = 1\cdot5 - \cdot4947 = 1\cdot0053$$

$$v = 2\cdot72 \times 1\cdot0053 = 2\cdot735 \text{ feet per second.}$$

Mean velocities—

(1)	By Kutter's formula	$v =$	4·4	feet per second.
(2)	" De Prony's formula	$v =$	2·79	"
(3)	" Dupuit's "	$v =$	2·78	"
(4)	" Chezy's "	$v =$	2·95	"
(5)	" Humphreys and Abbot's formula	$v =$	3·55	"
(6)	" Hagen's formula	$v =$	3·499	"
(7)	" De Prony's with Wiesbach's formula	$v =$	2·73	"
(8)	" Rankine's formula	$v =$	2·735	"
			8)25·434	
	Grand mean velocity	$v =$	3·18	"

To find the probable errors by the method of least squares, assuming each of the formulas to have equal weights.

No. of Formula.	Velocity.	Difference from mean.	Square of Residual errors.
1	4·4	— 1·22	1·4844
2	2·79	+ 0·39	0·1521
3	2·78	+ 0·40	0·1600
4	2·95	+ 0·23	0·0529
5	3·55	— 0·37	0·1369
6	3·499	— 0·32	0·1024
7	2·73	+ 0·48	0·2304
8	2·735	+ 0·445	0·1981
Mean =	3·18	$[n\,n_1]$ =	2·5172

Probable error :—

$$r = \cdot6745 \sqrt{\frac{[n\,n_1]}{m-1}} = \cdot6745 \sqrt{\frac{2\cdot5172}{8-1}}$$

$$= \cdot6745\ \sqrt{0\cdot3596} = \cdot6745 \times \pm 0\cdot6$$

$$= \pm\ 0\cdot415 \text{ feet.}$$

Probable error of arithmetical mean—

$$r\,(x) = \frac{\cdot6745}{\sqrt{m}} \sqrt{\frac{[n\,n_1]}{m-1}} = \frac{\cdot6745}{\sqrt{8}} \sqrt{\frac{5\cdot2172}{8-1}}$$

$$= \frac{\cdot6745}{2\cdot8284}\ \sqrt{0\cdot3596} = \cdot2385 \times \pm 0\cdot6$$

$$= \pm\ 0\cdot143 \text{ feet.}$$

Mean discharge, D = 3542 × 3·18 = 11,264 cubic feet per second.

" in earth section = 12,448 cubic feet per second.

" rock " = 11,335 "

Difference = 1,113

To discharge this additional quantity through the contracted channel, there must be an increased head of pressure at the contracted outlet. This is provided for not only through the declivity of 1·74 feet in the clay cut, but also by an increased depth of 2·45 feet at Robey St. Junction.

2ndly. Elevation of lake surface one foot above low water of November, 1895.

1st.—Channel in Earth.

CROSS-SECTION.

Width at bottom..... 202 feet.
Depth..... 23 "
Side slopes..... 2 horizontal to 1 vertical.
Width at water surface, (101+46)2=..... 294 feet.
Sectional area, $A = \frac{202 + 294}{2} \times 23 =$..... 5,704 square feet.
Wetted perimeter, $(101 + \sqrt{23^2+46^2})2 = (101+51{\cdot}5)\times 2 =$..... 305 feet.
Hydraulic mean radius, $m = \frac{5704}{305} =$..... 18·7 feet.
Slope of channel, $i = \frac{1}{40000} =$..... ·000025 feet per foot linear.

1.—*Kutter's Formula.*

Mean velocity—

$$v = \left\{ \frac{41{\cdot}6 + \frac{1{\cdot}811}{n} + \frac{{\cdot}00281}{i}}{1 + \left(41{\cdot}6 + \frac{{\cdot}00281}{i}\right)\frac{n}{\sqrt{m}}} \right\} \sqrt{mi}$$

$$= \left\{ \frac{41{\cdot}6 + \frac{1{\cdot}811}{{\cdot}0275} + \frac{{\cdot}00281}{{\cdot}000025}}{1 + \left(41{\cdot}6 + \frac{{\cdot}00281}{{\cdot}000025}\right)\frac{{\cdot}0275}{\sqrt{18{\cdot}7}}} \right\} \sqrt{18{\cdot}7 \times {\cdot}000025}$$

$$= \left(\frac{41{\cdot}6 + 65{\cdot}85 + 112{\cdot}4}{1 + (41{\cdot}6 + 112{\cdot}4) \times {\cdot}000636}\right) \times {\cdot}0216$$

$$= \frac{219{\cdot}85}{1{\cdot}98} \times {\cdot}0216$$

$$= 111 \times {\cdot}0216$$

$$= 2{\cdot}4 \text{ feet per second.}$$

2.—*de Prony's Formula.*

Mean velocity—

$$v = \left({\cdot}0237 + 9966mi\right)^{\frac{1}{2}} - {\cdot}1542$$

$$= \left({\cdot}0237 + 9966 \times 18{\cdot}7 \times {\cdot}000025\right)^{\frac{1}{2}} - {\cdot}1542$$

$$= \left({\cdot}0237 + 4{\cdot}6591\right)^{\frac{1}{2}} - {\cdot}1542$$

$$= \sqrt{4{\cdot}6828} - {\cdot}1542$$

$$= 2{\cdot}164 - 1542. \qquad = 2 \text{ feet per second.}$$

3.—*Dupuit's Formula.*

Mean velocity—

$${\cdot}0000323v^2 = i(1 + {\cdot}225m)$$

$$= (1 + {\cdot}225 \times 18.7) \times {\cdot}000025$$

$$= (1 + 4{\cdot}2125) \times {\cdot}000025$$

$$= 5{\cdot}2125 \times {\cdot}000025 = {\cdot}00013$$

Then $v^2 = \frac{{\cdot}00013}{{\cdot}0000323} = 4$; and

$v = 2$ feet per second.

4.—Chezy's Formula.

Mean velocity—

$$v = c\sqrt{mi}. \quad \text{Making } c = 100.$$

Then

$$v = 100\sqrt{18{\cdot}7 + {\cdot}000025}$$
$$= 100 \times {\cdot}0216$$
$$= 2{\cdot}16 \text{ feet per second.}$$

5.—Humphreys and Abbot's Formula.

Mean velocity—

$$v = \left\{\left(225\frac{m}{2}\sqrt{i}\right)^{\frac{1}{2}} - {\cdot}0388\right\}^2$$
$$= \left\{\left(225 \times \frac{18{\cdot}7}{2}\sqrt{{\cdot}000025}\right)^{\frac{1}{2}} - {\cdot}0388\right\}^2$$
$$= \left(\sqrt[4]{\frac{21{\cdot}038}{2}} - {\cdot}0388\right)^2$$
$$= \left(\sqrt[4]{10{\cdot}519} - {\cdot}0388\right)^2$$

No.	Log.
$10{\cdot}519$	$= 1{\cdot}0219745 \div 4 = 0{\cdot}2554937$
$1{\cdot}7927$	$= 0{\cdot}2554937$

$\therefore \quad v = (1{\cdot}7927 - {\cdot}0388)^2 = 1.754)^2$

$3{\cdot}07$ feet per second.

6.—Hagen's Formula.

Mean velocity—

$$v = 1\sqrt{m}\ \sqrt[6]{i} = 4{\cdot}39 \times \sqrt{18{\cdot}7} \times \sqrt[6]{{\cdot}000025}$$
$$= 4{\cdot}39 \times 4{\cdot}32 \times {\cdot}171$$
$$= 3{\cdot}24 \text{ feet per second.}$$

7.—de Prony's with Wiesbach's Coefficients.

Mean velocity—

$$v = \left({\cdot}00024 + 8695\, m\, i\right)^{\frac{1}{2}} - {\cdot}015$$
$$= \left({\cdot}00024 + 8695 \times 18{\cdot}7 \times {\cdot}000025\right)^{\frac{1}{2}} - {\cdot}015$$
$$= \left({\cdot}00024 + 4{\cdot}056\right)^{\frac{1}{2}} - {\cdot}015$$
$$= \sqrt{4{\cdot}0557} - {\cdot}015$$
$$= 2{\cdot}014 - {\cdot}015 = 2 \text{ feet per second.}$$

8.—Rankine's Formula.

Mean velocity—1st. Approximation—

$$v' = 8{\cdot}025\sqrt{\frac{mi}{{\cdot}007565}} = \sqrt{8512\, mi} = 92{\cdot}26\sqrt{mi}$$
$$= 92{\cdot}26\sqrt{18{\cdot}7 \times {\cdot}000025}$$
$$= 92{\cdot}26\sqrt{{\cdot}0004675}$$
$$= 92{\cdot}26 \times {\cdot}0216 = 1{\cdot}99 \text{ feet.}$$
$$f = \left({\cdot}0074 + \frac{{\cdot}00023}{1{\cdot}99}\right)$$
$$= ({\cdot}0074 + {\cdot}000116) = {\cdot}007516$$

Corrected value—

$$v = 1{\cdot}99\left(\frac{3}{2}=\frac{f}{{\cdot}01513}\right) = 1{\cdot}99\left(1{\cdot}5 - \frac{{\cdot}007516}{{\cdot}01513}\right)$$
$$= 1{\cdot}99\,(1{\cdot}5 - {\cdot}497) = 1{\cdot}99 \times 1{\cdot}003$$
$$= 1{\cdot}99 \text{ feet per second.}$$

Mean velocities.

(1)	By Kutter's formula	$v = 2{\cdot}4$	feet per second.
(2)	" de Prony's "	$v = 2{\cdot}0$	"
(3)	" Dupuit's "	$v = 2{\cdot}0$	"
(4)	" Chézy's "	$v = 2{\cdot}16$	"
(5)	" Humphreys and Abbot's formula	$v = 3{\cdot}07$	"
(6)	" Hagen's formula	$v = 3{\cdot}24$	"
(7)	" de Prony's with Wiesbach's co-efficients	$v = 2{\cdot}0$	"
(8)	" Rankine's formula	$v = 1{\cdot}99$	"
		$8\overline{)18{\cdot}86}$	
	Grand mean velocity	$v = 2{\cdot}36$	

To find the probable errors by the method of least squares, assuming each of the formulas to have equal weights.

No. of Formula.	Velocity.	Difference from Mean.	Square of Residual Error.
1	2·4	— 0·04	0·0016
2	2·0	+ 0·36	0·1296
3	2·0	+ 0·36	0·1296
4	2·16	+ 0·20	0·0400
5	3·07	— 0.71	0·5141
6	3·24	— 0·88	0·7744
7	2·0	+ 0·36	0·1296
8	1·99	+ 0·37	0·1369
	Mean = 2·36		$[nn_1] = 1{\cdot}8555$

Probable error—

$$r = {\cdot}6745\sqrt{\frac{[nn_1]}{m-1}} = {\cdot}6745\sqrt{\frac{1{\cdot}8556}{8-1}}$$
$$= {\cdot}6745\sqrt{0{\cdot}2651} = {\cdot}6745 \times \pm 0{\cdot}51$$
$$= \pm 0{\cdot}345 \text{ feet.}$$

Probable error of Arithmetical Mean—

$$r\,(x) = \frac{{\cdot}6745}{\sqrt{m}}\sqrt{\frac{[nn_1]}{m-1}} = \frac{{\cdot}6745}{\sqrt{8}}\sqrt{\frac{1{\cdot}8556}{8-1}}$$
$$= \frac{{\cdot}6745}{2{\cdot}8284}\sqrt{0{\cdot}2651} = {\cdot}2385 \times \pm 0{\cdot}51$$
$$= 0{\cdot}122 \text{ feet.}$$

Mean discharge, D = 5704 × 2·36 = 13,460 cubic feet per second.

2nd—Channel in Rock.

CROSS-SECTION.

Width at bottom	160 feet.
do surface	162 do
Mean width	161 do
Depth	23 do
Sectional area, A = 161 × 23	3,703 square feet.
Wetted perimeter = 160 + 23 × 2	206 feet.
Hydraulic mean radius, $m = \frac{3703}{206}$	18 do
Slope, i = 1 in 20,000	·00005 feet per foot linear.

1. *Kutter's Formula.*

Coefficient of roughness, $n = \cdot 017$, same as rubble.

Mean velocity—

$$v = \left\{ \frac{41\cdot6 + \frac{1\cdot811}{n} + \frac{\cdot00281}{i}}{1 + \left(41\cdot6 + \frac{\cdot00281}{i}\right)\frac{n}{\sqrt{m}}} \right\} \sqrt{m\,i}$$

$$= \left\{ \frac{41\cdot6 + \frac{1\cdot811}{\cdot017} + \frac{\cdot00281}{\cdot00005}}{1 + \left(41\cdot6 + \frac{\cdot00281}{\cdot00005}\right)\frac{\cdot017}{\sqrt{18}}} \right\} \sqrt{18 \times \cdot00005}$$

$$= \left\{ \frac{41\cdot6 + 106\cdot5 + 56\cdot2}{1 + \left(41\cdot6 + 56\right)\frac{\cdot017}{4\cdot24}} \right\} \sqrt{\cdot0009}$$

$$= \left(\frac{204\cdot3}{1 + 97\cdot6 \times \cdot004} \right) \sqrt{\cdot0009}$$

$$= \frac{204\cdot3 \times \cdot03}{1 + \cdot38} = \frac{6\cdot129}{1\cdot38}$$

$$= 4\cdot44 \text{ feet per second.}$$

2.—*DeProny's Formula.*

Mean velocity—

$$v = \left(\cdot0237 + 9966\,m\,i\right)^{\frac{1}{2}} - \cdot1542.$$

$$= \left(\cdot0237 + 9966 \times 18 \times \cdot00095\right)^{\frac{1}{2}} - \cdot1542.$$

$$= \left(\cdot0237 + 8\cdot87\right)^{\frac{1}{2}} - \cdot1542$$

$$= \sqrt{8\cdot8937} - \cdot1542$$

$$= 2\cdot9822 - \cdot1542$$

$$= 2\cdot828 \text{ feet per second.}$$

3.—*Dupuit's Formula.*

Mean velocity—

$$\cdot0000323\,v^2 = i\,(1 + \cdot225\,m) = (1 + \cdot225 \times 18) \times \cdot00005$$

$$= (1 + 4\cdot05) \times \cdot00005$$

$$= 5\cdot05 \times \cdot00005 = \cdot0002525$$

$$\therefore \quad v^2 = \frac{\cdot0002525}{\cdot0000323} = 7\cdot82$$

$$v = \sqrt{7\cdot82} = 2\cdot8 \text{ feet per second.}$$

4.—*Chezy's Formula.*

Mean velocity—

$$v = c\sqrt{m\,i}. \quad c \text{ is taken at } 100\cdot \text{ substitute.}$$

$$v = 100\sqrt{18 \times \cdot00005}$$

$$= 100\sqrt{\cdot009}$$

$$= 100 \times \cdot03$$

$$= 3 \text{ feet per second.}$$

5—Humphreys and Abbot's Formula.

Mean velocity—

$$v = \left\{\left(225\,\frac{m}{2}\,\sqrt{i}\right)^{\frac{1}{2}} - \cdot0388\right\}^2$$
$$= \left\{\left(225 \times \frac{18}{2}\,\sqrt{\cdot00005}\right)^{\frac{1}{2}} - \cdot0382\right\}^2$$
$$= \left\{\left(225 \times 9 \times \cdot007\right)^{\frac{1}{2}} - \cdot0388\right\}^2$$
$$= \left(\sqrt[4]{14\cdot175} - \cdot0388\right)^2$$
$$= \left(1\cdot34 - \cdot0388\right)^2$$
$$= \overline{1\cdot92}^2$$
$$= 3\cdot71 \text{ feet per second.}$$

6.—Hagen's Formula.

Mean velocity—

$$v = 4\cdot39\,\sqrt{m} \cdot \sqrt[6]{i}$$
$$= 4\cdot39\,\sqrt{18} \cdot \sqrt[6]{\cdot00005}$$
$$= 4\cdot39 \times 4\cdot24 \times \cdot191$$
$$= 3\cdot555 \text{ feet per second.}$$

7.—DeProny's Formula with Wiesbach's Coefficients.

Mean velocity—

$$v = \left(\cdot00024 + 8675\,m\,i\right)^{\frac{1}{2}} - \cdot015$$
$$= \left(\cdot00024 + 8675 \times 18 \times \cdot00005\right)^{\frac{1}{2}} - \cdot015$$
$$= \sqrt{7\cdot80774} - \cdot015$$
$$= 2\cdot794 - \cdot015$$
$$= 2\cdot78 \text{ feet per second.}$$

8.—Rankine's Formula.

Mean velocity—

$$v = 8\cdot025\sqrt{\frac{i\,m}{\cdot007565}} = \sqrt{8512\,i\,m} = 92\cdot26\,\sqrt{i\,m}$$
$$= 92\cdot26\,\sqrt{18 \times \cdot00005}$$
$$= 92\cdot26\,\sqrt{\cdot009}$$
$$= 92\cdot26 \times \cdot03$$
$$= 2\cdot768 \text{ feet per second.}$$

Mean velocities.

(1)	By Kutter's formula	$v = 4\cdot44$	feet per second.
(2)	De Prony's formula	$v = 2\cdot828$	"
(3)	Dupuit's "	$v = 2\cdot8$	"
(4)	Chezy's "	$v = 3\cdot0$	"
(5)	Humphrey's and Abbot's formula	$v = 3\cdot71$	"
(6)	Hagen's	$v = 3\cdot555$	"
(7)	De Prony's with Wiesbach's coefficients	$v = 2\cdot78$	"
(8)	Rankine's formulæ	$v = 2\cdot768$	"
		$8 \overline{)25\cdot881}$	
	Grand mean velocity	$v = 3\cdot24$	"

To find the probable errors by the method of least squares, assuming each formula to have equal weights.

Number of Formula.	Velocity.	Difference from Mean.	Square of Residuals.
1	4·44	— 1·20	1·4400
2	2·828	+ 0·412	0·1697
3	2·8	+ 0·44	0·1936
4	3·0	+ 0·24	0·0576
5	3·71	— 0·47	0·2209
6	3·555	— 0·315	0·0993
7	2·78	+ 0·46	0·2116
8	2·768	+ 0·472	0·2228
Mean	3·24		$[n\,n_1]$ 2·6155

Probable error—

$$r = \cdot 6745\sqrt{\frac{[n\,n_1]}{m-1}} = \cdot 6745\sqrt{\frac{2\cdot 6155}{8-1}}$$

$$= \cdot 6745\ \sqrt{0\cdot 4736} = \cdot 6745 \times \pm 0\cdot 61$$

$$= \pm 0\cdot 41 \text{ feet.}$$

Probable error of Arithmetical Mean—

$$r = \frac{\cdot 6745}{\sqrt{m}}\sqrt{\frac{[n\,n_1]}{m-1}} = \frac{\cdot 6745}{\sqrt{8}}\sqrt{\frac{2\cdot 6155}{8-1}}$$

$$= \frac{\cdot 6745}{2\cdot 8284}\ \sqrt{0\cdot 3736} = \cdot 2385 \times \pm 0\cdot 61$$

$$= \pm\ 0\cdot 145 \text{ feet.}$$

Discharge, $D = 3703 \times 3\cdot 24 = 12{,}000$ cubic feet per second.

3rdly.—Elevation of lake surface two feet above low water of November, 1895.

1ST.—CHANNEL IN EARTH.

CROSS-SECTION.

Width at bottom ..... 202 feet.
Depth ..... 24 "
Side slopes ..... 2 horizontal to 1 vertical.
Width at water surface ..... $(101 + 48) \times 2 = 298$ feet.
Sectional area A ..... $\frac{202 + 298}{2} \times 24 = 6{,}000$ square feet.
Slope ..... $i = 1$ in $40{,}000 = \cdot 000025$ feet per foot linear.
Wetted perimeter ..... $\left(101 + \sqrt{24^2 + 48^2}\right) \times 2$
$= (101 + 53\cdot 66) \times 2$
$= 154\ 66 \times 2 = 309\cdot 33$ feet.
Hydraulic mean radius ..... $m = \frac{6000}{309\cdot 3} = 19\cdot 4$ "

1.—Kutter's Formula.

Mean velocity—

$$v = \left\{\frac{41\cdot 6 + \frac{1\cdot 811}{n} + \frac{\cdot 00281}{i}}{1 + \left(41\cdot 6 + \frac{\cdot 00281}{i}\right)\frac{n}{\sqrt{m}}}\right\}\sqrt{mi}$$

$$= \left\{\frac{41\cdot 6 + \frac{1\cdot 811}{\cdot 0275} + \frac{\cdot 00201}{\cdot 000025}}{1 + \left(41\cdot 6 + \frac{\cdot 00281}{\cdot 000025}\right)\frac{\cdot 0275}{\sqrt{19\cdot 4}}}\right\}\sqrt{19\cdot 4 \times \cdot 000025}$$

$$= \left\{\frac{41\cdot 6 + 65\cdot 85 + 112\cdot 4}{1 + (41\cdot 6 + 112\cdot 4)\frac{\cdot 0275}{4\cdot 4}}\right\}\sqrt{\cdot 000485}$$

$$= \left(\frac{219\cdot 85}{1 + 154 \times \cdot 00625}\right) \times \cdot 021 = \frac{219\cdot 85}{1 + \cdot 9625} \times \cdot 021$$

$$= 112 \times \cdot 021 = 2\cdot 4 \text{ feet per second.}$$

2.—de Prony's Formula.

Mean velocity—

$$v = (\cdot 0237 + 9966\ mi)^{\frac{1}{2}} - \cdot 1542$$
$$= (\cdot 0237 + 9966 \times 19 \cdot 4 \times \cdot 000025)^{\frac{1}{2}} - \cdot 1542$$
$$= (\cdot 0237 + 4 \cdot 8335)^{\frac{1}{2}} - \cdot 1542$$
$$= \sqrt{4 \cdot 8572} - \cdot 1542$$
$$= 2 \cdot 2 - \cdot 1542 = 2 \cdot 04 \text{ feet per second.}$$

3.—Dupuit's Formula.

Mean velocity—

$$\cdot 0000323\ v^2 = i\ (1 + \cdot 225\ m)$$
$$= (1 + \cdot 225 \times 19 \cdot 4) \times \cdot 000025$$
$$= (1 + 4 \cdot 35) \times \cdot 000025$$
$$= 5 \cdot 35 \times \cdot 000025 = \cdot 000134$$
$$\therefore\ v^2 = \frac{\cdot 000134}{\cdot 0000323} = 4 \cdot 5$$
$$v = 2 \cdot 04 \text{ feet per second.}$$

4.—Chezy's Formula.

Mean velocity—

$$v = c \sqrt{mi}$$
$$= 100 \sqrt{19 \cdot 4 \times \cdot 000025}$$

$$v = 100 \sqrt{\cdot 000485}$$
$$= 100 \times \cdot 022 = 2 \cdot 2 \text{ feet per second.}$$

5.—Humphreys and Abbot's Formula.

Mean velocity—

$$v = \left\{ \left(225 \frac{m}{2} \sqrt{i} \right)^{\frac{1}{2}} - \cdot 0388 \right\}^2$$
$$= \left\{ \left(225 \times \frac{19 \cdot 4}{2} \sqrt{\cdot 000025} \right)^{\frac{1}{2}} - \cdot 0388 \right\}^2$$
$$= \left(\sqrt[4]{10 \cdot 6625} - \cdot 0388 \right)^2$$
$$= \left(1 \cdot 8 - \cdot 0488 \right)^2$$
$$= \left(1 \cdot 76 \right)^2 = 3 \cdot 1 \text{ feet per second.}$$

6.—Hagen's Formula

Mean velocity—

$$v = 4 \cdot 39 \sqrt{m} \cdot \sqrt[6]{i}$$
$$= 4 \cdot 39 \sqrt{19 \cdot 4} \cdot \sqrt[6]{\cdot 0000 - 25}$$
$$= 4 \cdot 39 \times 4 \cdot 4 \times \cdot 171$$
$$= 3 \cdot 3 \text{ feet per second.}$$

7.—De Prony's with Wiesbach's Coefficients.

Mean velocity—

$$v = \left(\cdot 00024 + 8675\ m\ i \right)^{\frac{1}{2}} - \cdot 015$$
$$= \left(\cdot 00024 + 8675 \times 19 \cdot 4 \times \cdot 000025 \right)^{\frac{1}{2}} - \cdot 015$$
$$= \left(\cdot 00024 + 4 \cdot 20738 \right)^{\frac{1}{2}} - \cdot 015$$
$$= \sqrt{4 \cdot 20762} - \cdot 015$$
$$= 2 \cdot 05 - \cdot 015$$
$$= 2 \cdot 035 \text{ feet per second.}$$

8.—Rankine's Formula.

Mean velocity—

$$v = 8{\cdot}025\sqrt{\frac{m\,i}{{\cdot}007565}} = \sqrt{8512\,mi} = 92{\cdot}26\ \sqrt{mi}$$

$$= 92{\cdot}26\ \sqrt{19{\cdot}4 + {\cdot}000025}$$

$$= 92{\cdot}26\ \sqrt{{\cdot}000485}$$

$$= 92{\cdot}26\ \sqrt{{\cdot}022} = 2{\cdot}02 \text{ feet per second.}$$

Mean velocities.

1.	By Kutter's formula	$v = 2{\cdot}4$	feet per second.
2.	" de Prony's "	$v = 2{\cdot}04$	"
3.	" Dupuit's "	$v = 2{\cdot}04$	"
4.	" Chezy's "	$v = 2{\cdot}2$	"
5.	" Humphreys and Abbot's formula	$v = 3{\cdot}1$	"
6.	" Hagen's formula	$v = 3{\cdot}3$	"
7.	" de Prony's with Wiesbach's Co-efficients	$v = 2{\cdot}035$	"
8.	" Rankine's Formula	$v = 2{\cdot}02$	"
		$8)\overline{19{\cdot}135}$	
	Grand mean velocity	$v = 2{\cdot}4$	

To find the probable errors by the method of least squares, assuming each of the formulas to have equal weight.

Number of Formula.	Velocity.	Difference from Mean.	Square of Residuals.
1	2·4	—0·00	0·0000
2	2·04	+0·36	0·1296
3	2·04	+0·36	0·1296
4	2·2	+0·2	0·0400
5	3·1	—0·7	0·4900
6	3·3	—0·9	0·8100
7	2·035	+0·365	0·1333
8	2·02	+0·38	0·1444
Mean =	2·4	$[n\,n_1]$ =	1·8769

Probable error—

$$r = {\cdot}6745\sqrt{\frac{[n\ n_1]}{m-1}} = {\cdot}6745\sqrt{\frac{1{\cdot}8769}{8-1}}$$

$$= {\cdot}6745\ \sqrt{0{\cdot}2681} = {\cdot}6745 \times \pm 0{\cdot}518$$

$$= \pm 0{\cdot}349 \text{ feet.}$$

Probable error of Arithmetical Mean—

$$r\,(x) = \frac{{\cdot}6745}{\sqrt{m}}\sqrt{\frac{[n\,n_1]}{m-1}} = \frac{{\cdot}6745}{\sqrt{8}}\sqrt{\frac{1{\cdot}8769}{8-1}}$$

$$= \frac{{\cdot}6745}{2{\cdot}8284}\sqrt{0{\cdot}2681}$$

$$= {\cdot}2385 \times \pm 0{\cdot}518 = \pm 0{\cdot}123 \text{ feet.}$$

Discharge, D = 6,000 × 2·4 = 14,400 cubic feet per second.

2nd. CHANNEL IN ROCK.

CROSS SECTION.

Width at bottom.....160 feet.
" water surface.....162 "
Mean width.....161 "
Depth.....24 "

Sectional area.....A = 161 × 24 = 3,864 square feet.
Wetted perimeter.....160 × 24 × 2 = 208 feet.
Hydraulic mean radius.....$m = \frac{3864}{208} = 18{\cdot}58$ "
Slope.....i = 1 in 20,000 = 00005 feet per foot linear.

1.—Kutter's Formula.

Co-efficient of roughness,=·017; same as rubble masonry.

Mean velocity—

$$v = \left\{ \frac{41{\cdot}6 + \frac{1{\cdot}811}{n} + \frac{{\cdot}00281}{i}}{1 \times \left(41{\cdot}6 + \frac{{\cdot}00281}{i}\right)\frac{n}{\sqrt{m}}} \right\} \sqrt{m\,i}$$

$$= \left\{ \frac{41{\cdot}6 + \frac{1{\cdot}811}{{\cdot}017} + \frac{{\cdot}00281}{{\cdot}00005}}{1 + \frac{1}{6}\left(41{\cdot}6 + \frac{{\cdot}00281}{{\cdot}00005}\right)\frac{{\cdot}017}{\sqrt{18{\cdot}58}}} \right\} \sqrt{18{\cdot}58 \times {\cdot}00005}$$

$$= \left\{ \frac{41{\cdot}6 + 106{\cdot}5 + 56{\cdot}2}{1 + (41{\cdot}6 + 56{\cdot}2)\frac{{\cdot}017}{4{\cdot}31}} \right\} \sqrt{{\cdot}000929}$$

$$= \frac{204{\cdot}3}{1 + 97{\cdot}6 \times {\cdot}004} \times {\cdot}0305$$

$$= \frac{204{\cdot}3 \times {\cdot}0305}{1 + {\cdot}39}$$

$$= \frac{6{\cdot}231}{1{\cdot}39}$$

$$= 4{\cdot}48 \text{ feet per second.}$$

2.—de Pronys Formula.

Mean velocity—

$$v = ({\cdot}0237 + 9966\,mi)^{\frac{1}{2}} - {\cdot}1542$$
$$= ({\cdot}0237 + 9966 \times 18{\cdot}58 \times {\cdot}00005)^{\frac{1}{2}} - {\cdot}1542$$
$$= ({\cdot}0237 + 9{\cdot}2584)^{\frac{1}{2}} - {\cdot}1542$$
$$= 3{\cdot}0466 - {\cdot}1542$$
$$= 2{\cdot}89 \text{ feet per second.}$$

3.—Dupuit's Formula.

Mean velocity—

$${\cdot}0000323\,v^2 = i(1 + {\cdot}225m)$$
$$= (1 + {\cdot}225 \times 18{\cdot}58) \times {\cdot}00005$$
$$= (1 + 4{\cdot}18) \times {\cdot}00005 = {\cdot}000259$$
$$\therefore \quad v^2 = \frac{{\cdot}000259}{{\cdot}0000323} = 8$$
$$v = 2{\cdot}83 \text{ feet per second.}$$

4.—Chezy's Formula.

Mean velocity—

$$v = c\sqrt{m\,i}, \quad c \text{ is taken} = 100, \text{ substitute.}$$
$$v = 100\sqrt{18{\cdot}58 \times {\cdot}00005}$$
$$= 100 \times {\cdot}0305$$
$$= 3{\cdot}05 \text{ feet per second.}$$

5.—Humphreys and Abbot's Formula.

Mean velocity—

$$v = \left\{ \left(225 \frac{m}{2} \sqrt{i} \right)^{\frac{1}{2}} - \cdot 0388 \right\}^2$$

$$= \left\{ \left(225 \times \frac{18 \cdot 58}{2} \sqrt{\cdot 00005} \right)^{\frac{1}{2}} - \cdot 0388 \right\}^2$$

$$= \left\{ \left(225 \times 9 \cdot 29 \times \cdot 007 \right)^{\frac{1}{2}} - \cdot 0388 \right\}^2$$

$$= \left\{ \sqrt[4]{14 \cdot 63175} - \cdot 0388 \right\}^2$$

$$= (1 \cdot 9545 - \cdot 0388)^2$$

$$= \overline{1 \cdot 916}\,)^2 = 3 \cdot 95 \text{ feet per second.}$$

6.—Hagen's Formula.

Mean velocity—

$$v = 4 \cdot 39 \sqrt{m} \sqrt[6]{i}$$

$$= 4 \cdot 39 \sqrt{18 \cdot 58} \sqrt[6]{\cdot 00005}$$

$$= 4 \cdot 39 \times 4 \cdot 31 \times \cdot 191$$

$$= 3 \cdot 614 \text{ feet per second.}$$

7.—de Prony's Formula with Wiesbach's Coefficients.

Mean velocity—

$$v = \left(\cdot 00024 + 8675\ m\ i \right)^{\frac{1}{2}} - \cdot 015$$

$$= \left(\cdot 00024 + 8675 \times 19 \cdot 58 \times \cdot 00005 \right)^{\frac{1}{2}} - \cdot 015$$

$$= \sqrt{8 \cdot 06} - \cdot 015$$

$$= 2 \cdot 84 - \cdot 015$$

$$= 2 \cdot 825 \text{ feet per second.}$$

8.—Rankine's Formula.

Mean velocity—

$$v = 8 \cdot 025 \sqrt{\frac{i\,m}{\cdot 007565}} = \sqrt{8512\ i\,m} = 92 \cdot 26 \sqrt{i\,m}$$

$$= 92 \cdot 26 \sqrt{18 \cdot 58 \times \cdot 00005}$$

$$= 92 \cdot 26 \sqrt{\cdot 000929}$$

$$= 92 \cdot 26 \times \cdot 0305 = 2 \cdot 84 \text{ feet per second.}$$

Mean velocities.

(1)	By Kutter's formula	$v = 4 \cdot 48$	feet per second.
(2)	Prony's "	$v = 2 \cdot 89$	"
(3)	Dupuit's "	$v = 2 \cdot 83$	"
(4)	Chezy's "	$v = 3 \cdot 05$	"
(5)	Humphreys & Abbot's formula	$v = 3 \cdot 95$	"
(6)	Hagen's formula	$v = 3 \cdot 614$	"
(7)	de Prony's with Wiesbach's coefficients	$v = 2 \cdot 825$	"
(8)	Rankine's formula	$v = 2 \cdot 814$	"
		$8\ )\ 26 \cdot 453$	
	Grand mean velocity	$v = 3 \cdot 31$	"

To find the probable errors by the method of least squares, assuming each of the formulas to have equal weights :—

No. of Formula.	Velocity.	Difference from mean.	Square of Residuals.
1	4·48	−1·17	1·3689
2	2·89	+ 0·42	0·1764
3	2·83	+ 0·48	0·2304
4	3·05	+ 0·26	0·0676
5	3·95	− 0·64	0·4096
6	3·614	− 0·304	0·0924
7	2·825	+ 0·485	0·2352
8	2·814	+ 0·496	0·2461
	Mean = 3·31		$[nn_1] = 2 \cdot 8266$

Probable error :—

$$r = \cdot 6745 \sqrt{\frac{[nn_1]}{m-1}} = \cdot 6745 \sqrt{\frac{2 \cdot 8266}{8-1}}$$
$$= \cdot 6745 \sqrt{\cdot 04038} = \cdot 6945 \times \pm 0 \cdot 63$$
$$= \pm 0 \cdot 425 \text{ feet.}$$

Probable error of arithmetical mean :—

$$r(x) = \frac{\cdot 6745}{\sqrt{m}} \sqrt{\frac{[nn_1]}{m-1}} = \frac{\cdot 6745}{\sqrt{8}} \sqrt{\frac{2.8266}{8-1}}$$
$$= \frac{\cdot 6745}{2 \cdot 8284} \sqrt{0 \cdot 4038}$$
$$= 2385 \times \pm 0 \cdot 63 = \pm 0 \cdot 15 \text{ feet.}$$

Mean discharge, $D = 3864 \times 3 \cdot 31 = 12{,}790$ cubic feet per second.

4thly.—Elevation of lake surface three feet above low water of November, 1895.

1.—CHANNEL IN EARTH.

CROSS-SECTION.

Width at bottom ..... 202 feet.
Depth. ..... 25 "
Side slopes ..... 2 horizontal to 1 vertical.
Width at water surface, (101+50)2 ..... 302 feet.

Sectional area, $A = \frac{202+303}{2} \times 25$ ..... 6,300 square feet.

Wetted perimeter, $\left(101 + \sqrt{25^2+50^2}\right) \times 2$
$= \left(101 + \sqrt{3125}\right) \times 2$
$= \left(101+56\right) \times 2$ ..... 314 feet.

Hydraulic mean radius, $m = \frac{6300}{314}$ ..... 20 "

Slope, $i = 1$ in $40{,}000 = \cdot 000025$ feet per foot linear.

1.—*Kutter's Formula.*

Mean velocity—

$$v = \left\{ \frac{41 \cdot 6 + \frac{1 \cdot 811}{n} + \frac{\cdot 00281}{i}}{1 + \left(41 \cdot 6 + \frac{\cdot 00281}{i}\right)\frac{n}{\sqrt{m}}} \right\} \sqrt{m\,i}$$
$$= \left\{ \frac{41 \cdot 6 + \frac{1 \cdot 811}{0 \cdot 275} + \frac{\cdot 00281}{\cdot 00025}}{1 + \left(41 \cdot 6 + \frac{\cdot 00281}{\cdot 000025}\right)\frac{\cdot 0275}{\sqrt{20}}} \right\} \sqrt{20 \times \cdot 000025}$$
$$= \left\{ \frac{41 \cdot 6 + 65 \cdot 85 + 112 \cdot 4}{1 + \left(41 \cdot 6 + 112 \cdot 5\right)\frac{\cdot 0275}{4 \cdot 47}} \right\} \sqrt{\cdot 0005}$$
$$= \left(\frac{219 \cdot 85}{1 + 154 \times \cdot 006}\right) \times \cdot 0224$$
$$= 114 \times \cdot 0224$$
$$= 2 \cdot 6 \text{ feet per second.}$$

2.—*de Prony's Formula.*

Mean velocity—

$$v = (\cdot 0237 + 9966\, m\, i)^{\frac{1}{2}} - \cdot 1542$$
$$= (\cdot 0237 + 9966 \times 20 \times \cdot 00(025)^{\frac{1}{2}} - \cdot 1542$$
$$= (\cdot 0237 + 4 \cdot 983)^{\frac{1}{2}} - \cdot 1542$$
$$= \sqrt{5} - \cdot 1542$$
$$= 2 \cdot 236 - \cdot 1542 = 2 \cdot 08 \text{ feet per second.}$$

3.—Dupuit's Formula.

Mean velocity—

$$\cdot 0000323\, v^2 = i\,(1 + \cdot 225\, m)$$
$$= (1 + \cdot 225 \times 20) \times \cdot 000025$$
$$= (1 + 4 \cdot 5) \times \cdot 000025$$
$$= 5 \cdot 5 \times \cdot 000025$$
$$= \cdot 0001375$$
$$\therefore v^2 = \frac{\cdot 0001375}{\cdot 0000323} = 4 \cdot 26$$
$$v = \sqrt{4.26} = 2 \cdot 60 \text{ feet per second.}$$

4.—Chezy's Formula.

Mean velocity—

$$v = c\sqrt{m\,i}$$
$$= 100\sqrt{20 \times 0000 \cdot 25}$$
$$= 100 \times \cdot 03236$$
$$= 2 \cdot 236 \text{ feet per second.}$$

5.—Humphreys and Abbot's Formula.

Mean velocity—

$$v = \left\{\left(225\frac{m}{2}\sqrt{i}\right)^{\frac{1}{2}} - \cdot 0388\right\}^2$$
$$= \left\{\left(225 \times \frac{20}{2}\sqrt{\cdot 000025}\right)^{\frac{1}{2}} - \cdot 0388\right\}^2$$
$$= \left(\sqrt[4]{11 \cdot 25} - \cdot 0388\right)^2$$
$$\text{Log. } 11 \cdot 25 = 1 \cdot 0511525 \div 4$$
$$1 \cdot 8314 = 0 \cdot 2627881$$
$$v = (1\ 8314 - 0 \cdot 388)^2 =$$
$$= \overline{1 \cdot 79})^2 = 3 \cdot 024 \text{ feet per second.}$$

6.—Hagen's Formula.

Mean velocity—

$$v = 4 \cdot 39\sqrt{m}\cdot\sqrt[6]{i}$$
$$= 4 \cdot 39\sqrt{20}\cdot\sqrt[6]{\cdot 000025}$$
$$= 4 \cdot 39 \times 4 \cdot 47 \times \cdot 171$$
$$= 3 \cdot 356 \text{ feet per second.}$$

7.—de Prony's with Wiesbach's Coefficients.

Mean velocity—

$$v = \left(\cdot 00024 + 8675\, m\, i\right)^{\frac{1}{2}} - \cdot 015$$
$$= \left(\cdot 00024 + 8675 \times 20 \times \cdot 000025\right)^{\frac{1}{2}} - \cdot 015$$
$$= \left(00024 + 4 \cdot 3375\right)^{\frac{1}{2}} - \cdot 015$$
$$= \sqrt{4 \cdot 33774} - \cdot 015$$
$$= 2 \cdot 083 - \cdot 015$$
$$= 2 \cdot 068 \text{ feet per second.}$$

8.—Rankine's Formula.

Mean velocity—

$$v = 8{\cdot}025\sqrt{\frac{m\,i}{{\cdot}007565}} = \sqrt{8512\ m\ i} = 92{\cdot}26\ \sqrt{m\,i}$$

$$= 9{\cdot}226\ \sqrt{20\ \times\ {\cdot}00025} = 92{\cdot}26\ \times\ {\cdot}02236$$

$$= 2{\cdot}06 \text{ feet per second.}$$

Mean velocities.

(1)	By Kutter's formula	$v = 2{\cdot}6$	feet per second.
(2)	deProny's "	$v = 2{\cdot}08$	"
(3)	Dupuit's "	$v = 2{\cdot}06$	"
(4)	Chezy's "	$v = 2{\cdot}236$	"
(5)	Humphreys and Abbot's formula	$v = 3{\cdot}204$	"
(6)	Hagen's formula	$v = 3{\cdot}356$	"
(7)	deProny's with Wiesbach's Coefficients	$v = 2{\cdot}068$	"
(8)	Rankine's formula	$v = 2{\cdot}06$	"
		$8)\overline{19{\cdot}664}$	
	Grand mean velocity	$v = 2{\cdot}458$	"

To find the probable errors by the method of least squares, assuming each of the formulas to have equal weights.

No. of Formula.	Velocity.	Difference from Mean.	Square of Residuals.
1	2·6	−0·142	0·0202
2	2·08	+0·378	0·1429
3	2·06	+0·398	0·1584
4	2·236	+0·222	0·0493
5	3·204	−0·746	0·5565
6	3·356	−0·898	0·8064
7	2·068	+0·390	0·154
8	2 06	+0·398	0·1584
	Mean = 2·458		$[nn_1] = 2{\cdot}0442$

Probable error—

$$r = {\cdot}6745\sqrt{\frac{[nn_1]}{m-1}} = {\cdot}6745\sqrt{\frac{2{\cdot}0442}{8-1}}$$

$$= {\cdot}6745\ \sqrt{0{\cdot}2920} = {\cdot}6745\ \times \pm 0{\cdot}54$$

$$= \pm 0{\cdot}36 \text{ feet per second.}$$

Probable error of arithmetical mean—

$$r\,(x) = \frac{{\cdot}6745}{\sqrt{m}}\sqrt{\frac{[nn_1]}{m-1}} = \frac{{\cdot}6745}{\sqrt{8}}\sqrt{\frac{2{\cdot}0442}{8-1}}$$

$$= \frac{{\cdot}6745}{2{\cdot}8284}\sqrt{0{\cdot}2920} = {\cdot}2385\ \times\ \pm\ 0{\cdot}54$$

$$= 0{\cdot}129 \text{ feet.}$$

Mean discharge, D. = 6300 × 2·458 = 15,480 cubic feet per second.

5thly.—Elevation of lake surface four feet above low water of November, 1895.

1ST.—CHANNEL IN EARTH.

CROSS-SECTION.

Width at bottom ..... 202 feet.
Depth ..... 26 "
Side slopes ..... 2 horizontal to 1 vertical.
Width at surface (101 + 52) × 2 ..... 306 feet.

Sectional area, $A = \frac{202 + 306}{2} \times 26 = 6{,}604$ square feet.

$$\text{Wetted perimeter } \left(101 + \sqrt{26^2 + 52^2}\right) \times 2$$

$$= \left(101 + \sqrt{3380}\right) \times 2$$

$$= (101 + 58) \times 2$$

$$= 159 \times 2 = 318 \text{ feet.}$$

$$\text{Hydraulic mean radius, } m = \frac{6604}{318} = 20{\cdot}77 \text{ feet.}$$

Slope, i = 1 in 40,000 = ·000025 feet per foot linear.

1—*Kutter's Formula.*

Mean velocity—

$$v = \left\{ \frac{41{\cdot}6 + \frac{1{\cdot}811}{n} + \frac{{\cdot}00281}{i}}{1 + \left(41\ 6 + \frac{{\cdot}00281}{i}\right)\frac{n}{\sqrt{m}}} \right\} \sqrt{m\,i}$$

$$= \left\{ \frac{41{\cdot}6 + \frac{1{\cdot}811}{{\cdot}0275} + \frac{{\cdot}00281}{{\cdot}000025}}{1 + \left(41{\cdot}6 + \frac{{\cdot}00281}{{\cdot}000025}\right)\frac{{\cdot}0275}{\sqrt{20{\cdot}77}}} \right\} \sqrt{20{\cdot}77 \times {\cdot}000025}$$

$$= \left\{ \frac{41{\cdot}6 + 65{\cdot}85 + 112{\cdot}4}{1 + \left(41{\cdot}6 + 112{\cdot}4\right)\frac{{\cdot}0275}{4{\cdot}557}} \right\} \sqrt{{\cdot}00052}$$

$$= \frac{219{\cdot}85}{1 + {\cdot}927} \times {\cdot}023$$

$$= \frac{219{\cdot}85 \times {\cdot}023}{1{\cdot}927}$$

$$= 2{\cdot}62 \text{ feet per second.}$$

2.—*DeProny's Formula.*

Mean velocity—

$$v = \left({\cdot}0237 + 9966\ m\ i\right)^{\frac{1}{2}} - {\cdot}1542$$

$$= \left({\cdot}0237 + 9966 \times 20{\cdot}77 \times {\cdot}000025\ \right)^{\frac{1}{2}} - {\cdot}1542$$

$$= \left({\cdot}0237 + 5{\cdot}175\ \right)^{\frac{1}{2}} - {\cdot}1542$$

$$= 2{\cdot}275 - {\cdot}1542$$

$$= 2{\cdot}12 \text{ feet per second.}$$

3.—*Dupuit's Formula.*

Mean velocity—

$${\cdot}0000323\ v^2 = i\,(1 + {\cdot}225\ m)$$

$$= (1 + {\cdot}225 \times 20{\cdot}77) \times {\cdot}000025$$

$$= (1 + 4{\cdot}67) \times {\cdot}000025$$

$$= 5{\cdot}67 \times {\cdot}000025$$

$$= {\cdot}000142$$

$$\therefore\ v^2 = \frac{{\cdot}000142}{{\cdot}0000323} = 4{\cdot}4$$

$$v = 2{\cdot}1 \text{ feet per second.}$$

4.—*Chezy's Formula.*

$$v = c\ \sqrt{m\,i}$$

$$= 100\ \sqrt{20{\cdot}77 \times {\cdot}000025}$$

$$= 100 \times {\cdot}023$$

$$= 2{\cdot}3 \text{ feet per second.}$$

5.—*Humphrey's and Abbot's Formula.*

Mean velocity—

$$v = \left\{ \left(225\ \frac{m}{2}\ \sqrt{i}\ \right)^{\frac{1}{4}} - {\cdot}0388 \right\}^2$$

$$v = \left\{ \left(225 \times \frac{20{\cdot}77}{2}\ \sqrt{{\cdot}000025}\ \right)^{\frac{1}{4}} - {\cdot}0388 \right\}^2$$

$$= \left\{ \left(\frac{4673{\cdot}25 \times {\cdot}005}{2}\ \right)^{\frac{1}{4}} - {\cdot}0388 \right\}^2$$

$$= \left(\sqrt[4]{11{\cdot}683}\ - {\cdot}0388\right)^2$$

$$\text{Log.} = 11{\cdot}683 = 1{\cdot}0675544 \div 4$$

$$= 1{\cdot}8488 = 0{\cdot}2668886$$

$$v = (1{\cdot}8488 - {\cdot}0388)^2 = \overline{1{\cdot}81})^2 = 3{\cdot}27 \text{ feet per second.}$$

6.—Hagen's Formula.

Mean velocity—

$$v = 4{\cdot}39 \sqrt{m} \sqrt[6]{i}$$
$$= 4{\cdot}39 \times 4{\cdot}557 \times {\cdot}171$$
$$= 3{\cdot}42 \text{ feet per second.}$$

7.—De Prony's with Wiesbach's Coefficients.

Mean velocity—

$$v = \left({\cdot}00024 + 8675\ m\ i\right)^{\frac{1}{2}} - {\cdot}015$$
$$= \left({\cdot}00024 + 8675 \times 20{\cdot}77 \times {\cdot}000025\right)^{\frac{1}{2}} - {\cdot}015$$
$$= \sqrt{4{\cdot}5} - {\cdot}015$$
$$= 2{\cdot}122 - {\cdot}015$$
$$= 2{\cdot}1 \text{ feet per second.}$$

8.—Rankine's Formula.

Mean velocity—

$$v = 8{\cdot}025 \sqrt{\frac{m\ i}{{\cdot}007565}} = \sqrt{8512\ m\ i} = 92{\cdot}26 \sqrt{m\ i}$$
$$= 92{\cdot}26 \sqrt{20{\cdot}77 \times {\cdot}000025}$$
$$= 92{\cdot}26 \times {\cdot}023 = 2{\cdot}122 \text{ feet per second.}$$

Mean Velocities.

(1)	By Kutter's formula	v =	2·62	feet per second.
(2)	" De Prony's formula	v =	2·12	"
(3)	" Dupuit's "	v =	2·1	"
(4)	" Chezy's "	v =	2·3	"
(5)	" Humphrey's and Abbot's formula	v =	3·276	"
(6)	" Hagen's formula	v =	3·42	"
(7)	" De Prony's with Wiesbach's coefficients	v =	2·1	"
(8)	" Rankine's formula	v =	2·122	"
			8)20·058	
	Grand mean velocity.	v =	2·507	"

To find the probable errors by the method of least squares, assuming each of the formulas to have equal weights :—

No. of Formula.	Velocity.	Difference from Mean.	Square of Residuals.
1	2·62	−0·113	0·0128
2	2·12	+0·387	0·1498
3	2·1	+0·407	0·1656
4	2·3	+0·207	0·0429
5	3·276	−0·769	0·5914
6	3·42	−0·873	0·7621
7	2·1	+0·407	0·1657
8	2·122	+0·385	0·1482
	Mean = 2·507		$[nn_1]$ = 2·0385

Probable error—

$$r = {\cdot}6745 \sqrt{\frac{[nn_1]}{m-1}} = {\cdot}6745 \sqrt{\frac{2{\cdot}0385}{8-1}}$$
$$= {\cdot}6745 \sqrt{0{\cdot}2912} = {\cdot}6745 \times \pm 0{\cdot}54$$
$$= \pm 0{\cdot}36 \text{ feet.}$$

Probable error of arithmentical mean—

$$r\,(x) = \frac{{\cdot}6745}{\sqrt{m}} \sqrt{\frac{[nn_1]}{m-1}} = \frac{{\cdot}6745}{\sqrt{8}} \sqrt{\frac{2.0385}{8-1}}$$
$$= \frac{{\cdot}6745}{2{\cdot}8284} \sqrt{0{\cdot}2912} = {\cdot}2385 \times \pm 0{\cdot}54$$
$$= \pm 0{\cdot}129 \text{ feet.}$$

Mean discharge, D = 6604 × 2·507 = 16,560 cubic feet per second

6thly.—Elevation of lake surface five feet above low water of November, 1895.

CHANNEL IN EARTH.

Width at bottom..... 202 feet.
Depth..... 27 "
Side slopes..... 2 horizontal to 1 vertical.
Width at water surface..... $(101 + 54) \times 2 = 310$ feet.

Sectional area..... $A = \frac{202 + 310}{2} \times 27 = 6{,}912$ square miles.

Wetted perimeter..... $(101 + \sqrt{27^2 + 54^2}) \times 2 = 322{\cdot}75$ feet.

Hydraulic mean radius..... $m = \frac{6912}{322{\cdot}75} = 21{\cdot}4$ "

Slope of channel..... $i = 1$ in $40{,}000 = {\cdot}000025$ feet per foot linear.

1.—Kutter's Formula.

$$v = \left\{ \frac{41{\cdot}6 + \frac{1{\cdot}811}{n} + \frac{{\cdot}00281}{i}}{1 + \left(41{\cdot}6 + \frac{{\cdot}00281}{i}\right)\frac{n}{\sqrt{m}}} \right\} \sqrt{m\,i}$$

$$= \left\{ \frac{41{\cdot}6 + \frac{1{\cdot}811}{{\cdot}0275} + \frac{{\cdot}00281}{{\cdot}000025}}{1 + \left(41{\cdot}6 + \frac{{\cdot}00281}{{\cdot}000025}\right)\frac{{\cdot}0275}{\sqrt{21{\cdot}4}}} \right\} \sqrt{21{\cdot}4 \times {\cdot}000025}$$

$$v = \left\{ \frac{41{\cdot}6 + 65{\cdot}85 + 112{\cdot}4}{1 + \left(41{\cdot}6 + 112{\cdot}4\right)\frac{{\cdot}0275}{4{\cdot}626}} \right\} \sqrt{{\cdot}000535}$$

$$= \left(\frac{219{\cdot}85}{1 + 154 \times \frac{{\cdot}0275}{4{\cdot}626}} \right) \sqrt{{\cdot}000535}$$

$$= \frac{219{\cdot}85}{1 + {\cdot}916} \times {\cdot}02313$$

$$= 114{\cdot}75 \times {\cdot}02313$$

$$= 2{\cdot}65 \text{ feet per second.}$$

2.—de Prony's Formula.

Mean velocity –

$$v = \left({\cdot}0237 + 9966\, m\, i\right)^{\frac{1}{2}} - {\cdot}1542$$

$$= \left({\cdot}0237 + 9966 \times 21{\cdot}4 \times {\cdot}000025\right)^{\frac{1}{2}} - {\cdot}1542$$

$$= \left({\cdot}0237 + 5{\cdot}3318\right)^{\frac{1}{2}} - {\cdot}1542$$

$$= \sqrt{5{\cdot}3555} - {\cdot}1542$$

$$= 2{\cdot}3154 - {\cdot}1542$$

$$= 2{\cdot}16 \text{ feet per second.}$$

3.—Dupuit's Formula.

Mean velocity—

$${\cdot}0000323\, v^2 = i\,(1 + {\cdot}225\, m)$$

$$= (1 + {\cdot}225 \times 21{\cdot}4) \times {\cdot}000025$$

$$= (1 + 4{\cdot}815) \times {\cdot}000025$$

$$= {\cdot}0001454$$

$$\therefore v^2 = \frac{{\cdot}0001454}{{\cdot}0000323} = 4{\cdot}5$$

$$v = 2{\cdot}11 \text{ feet per second.}$$

4.—Chezy's Formula.

Mean velocity—

$$v = c\sqrt{m\,i}$$

$$= 100\sqrt{21{\cdot}4 \times {\cdot}000025}$$

$$= 100\sqrt{{\cdot}000535}$$

$$= 100 \times {\cdot}02313$$

$$= 2{\cdot}313 \text{ feet per second.}$$

5.—Humphreys and Abbot's Formula.

Mean velocity—

$$v = \left\{\left(225\ \frac{m}{2}\ \sqrt{i}\right)^{\frac{1}{2}} - \cdot0388\right\}^2$$

$$= \left\{\left(225 \times \frac{21\cdot4}{2}\ \sqrt{\cdot000025}\right)^{\frac{1}{2}} - \cdot0288\right\}^2$$

$$= \left(\sqrt[4]{12\cdot0375} - \cdot0388\right)^2$$

$$\log.\ 12\cdot038 = 1\cdot0805543 \div 4$$

$$1\cdot8628 = 0\cdot2701386$$

$$\therefore v = (1\cdot8628 - \cdot0388)^2$$

$$= \overline{1\cdot824})^2$$

$$= 3\cdot327 \text{ feet per second.}$$

6.—Hagen's Formula.

Mean velocity—

$$v = 4\cdot39\ \sqrt{m}\ \sqrt[6]{i}$$

$$= 4\cdot39 \times \sqrt{21\cdot4}\ \sqrt[6]{\cdot000025}$$

$$= 4\cdot39 \times 4\cdot626 \times \cdot171$$

$$= 3\cdot473 \text{ feet per second.}$$

7.—de Prony's with Wiesbach's Coefficients.

Mean velocity—

$$v = \left(\cdot00024 + 8675\ m\ i\right)^{\frac{1}{2}} - \cdot015$$

$$= \left(\cdot00024 + 8675 \times 21\cdot4 \times \cdot000025\right)^{\frac{1}{2}} - \cdot015$$

$$v = (\cdot00024 + 6\cdot6414)^{\frac{1}{2}} - \cdot015.$$

$$= \sqrt{4\cdot6414} - \cdot015 = 2\cdot154 - \cdot015.$$

$$= 2\cdot14 \text{ feet per second.}$$

8. Rankine's Formula.

Mean velocity—

$$v = 8\cdot025 \sqrt{\frac{m\,i}{\cdot007565}} = \sqrt{8512\ mi} = 92\cdot26\ \sqrt{m\,i}$$

$$= 92\cdot26\ \sqrt{21\cdot4 \times \cdot000025}$$

$$= 92\cdot26\ \sqrt{\cdot00535}$$

$$= 92\cdot26 \times \cdot02315$$

$$= 2\cdot136 \text{ feet per second.}$$

Mean velocities.

1.	By Kutter's formula	$v = 2\cdot65$	feet per second.
2.	" de Prony's "	$v = 2\cdot16$	"
3.	" Dupuit's "	$v = 2\cdot11$	"
4.	" Chezy's "	$v = 2\cdot313$	"
5.	" Humphreys and Abbot's formula	$v = 3\cdot327$	"
6.	" Hagen's formula	$v = 3\cdot473$	"
7.	" de Prony's with Wiesbach's Co-efficients	$v = 2\cdot14$	"
8.	" Rankine's formula	$v = 2\cdot136$	"
		$8)\overline{20\cdot309}$	
	Grand mean velocity	$v = 2\cdot538$	

To find the probable errors by the method of least squares, assuming each of the formulas to have equal weights.

Number of Formula.	Velocity.	Difference from Mean.	Square of Residuals.
1	2·65	—0·112	0·0125
2	2·16	+0·378	0·1429
3	2·11	+0·428	0·1832
4	2·313	+0·225	0·0506
5	3 327	—0·789	0·6225
6	3·473	—0·935	0·8742
7	2·14	+0·398	0·1584
8	2·136	+0·402	0·1616
Mean =	2·538	$[n\ n_1]$ =	2·2059

Probable error—

$$r\,(x) = \cdot 6745 \sqrt{\frac{[n\ n_1]}{m-1}} = \cdot 6745 \sqrt{\frac{2\cdot 2059}{8-1}}$$

$$= \cdot 6745 \sqrt{0\cdot 3123} = \cdot 6745 \times \pm 0\cdot 56.$$

$$= \pm 0\cdot 38 \text{ feet.}$$

Probable error of Arithmetical Mean—

$$r\,(x) = \frac{6745}{\sqrt{m}} \sqrt{\frac{[n\ n_1]}{m-1}} = \frac{\cdot 6745}{\sqrt{8}} \sqrt{\frac{2\cdot 2059}{8-1}}$$

$$= \frac{\cdot 6745}{2\cdot 8284} \sqrt{0\cdot 3123} = \cdot 2385 \times \pm 0\cdot 56$$

$$= \pm 0\cdot 133 \text{ feet.}$$

Mean Discharge, D = 6912 × 2·538 = 17,540 cubic feet per second.

The section of the channel in earth extends from Robey Street junction 13·15 miles or 69,432 feet. The fall in this distance, at ·000025 feet per foot linear, is 1·75 feet nearly.

2ND.—CHANNEL IN ROCK.

CROSS SECTION.

Width at bottom	160 feet.
" water surface	162 "
Mean width	161 "
Depth	23·75 "
Sectional area	$A = 161 \times 23\cdot 75 = 3823\cdot 75$ square "
Wetted perimeter	$160 \times 23\cdot 75 = 207\cdot 5$ "
Hydraulic mean radius	$m = \frac{3823\cdot 75}{207\cdot 5} = 18\cdot 42$ "
Slope	$i = 1$ in 20,000 = ·00005 feet per foot linear.

1.—Kutter's Formula.

Co-efficient of roughness, $n = \cdot 017$; same as for rubble.

Mean velocity—

$$v = \left\{ \frac{41\cdot6 + \frac{1\cdot811}{n} + \frac{\cdot00281}{i}}{1 + \left(41\cdot6 + \frac{\cdot00281}{i}\right)\frac{n}{\sqrt{m}}} \right\} \sqrt{m\,i}$$

$$= \left\{ \frac{41\cdot6 + \frac{1\cdot811}{\cdot017} + \frac{\cdot00281}{\cdot00005}}{1 + \left(41\cdot6 + \frac{\cdot00281}{\cdot00005}\right)\frac{\cdot017}{\sqrt{18\cdot42}}} \right\} \sqrt{18\cdot42 \times \cdot00005}$$

$$= \left\{ \frac{41\cdot6 + 106\cdot5 + 56\cdot2}{1 + (41\cdot6 + 56)\frac{\cdot017}{4\cdot29}} \right\} \sqrt{\cdot000951}$$

$$= \left(\frac{204\cdot3}{1 + 97\cdot6 + \cdot004} \right) \times \cdot0303$$

$$= \frac{204\cdot + \cdot0303}{1 + \cdot39} = \frac{6\cdot19}{1\cdot39}$$

$= 4\cdot455$ feet per second.

2.—de Prony's Formula.

Mean velocity—

$$v = \left(\cdot0237 + 9966\,m\,i\right)^{\frac{1}{2}} - \cdot1542$$

$$= \left(\cdot0237 + 9966 \times 18\cdot42 \times \cdot00005\right)^{\frac{1}{2}} - \cdot1542$$

$$= \left(\cdot0237 + 9\cdot1787\right)^{\frac{1}{2}} - \cdot1542$$

$$= \sqrt{9\cdot2024} - \cdot1542 = 3\cdot0335 - \cdot1542$$

$= 2\cdot879$ feet per second.

3.—Dupuit's Formula.

Mean velocity—

$$\cdot0000323\,v^2 = i\,(1 + \cdot225\,m)$$

$$= (1 + \cdot225 \times 18\cdot42) \times \cdot00005$$

$$= (1 + 4\cdot145) \times 00005 = \cdot0008725$$

$$\therefore \quad v^2 = \frac{\cdot00025725}{\cdot0000323} = 8$$

$v = 2\cdot83$ feet per second.

3.—Chezy's Formula.

Mean velocity—

$$v = c\,\sqrt{m\,i}$$

$$= 100\,\sqrt{18\cdot42 \times \cdot00005}$$

$$= 100 \times \cdot0303$$

$= 3\cdot03$ feet per second.

5.—Humphreys and Abbot's Formula.

Mean velocity—

$$v = \left\{ \left(225\frac{m}{2}\sqrt{i}\right)^{\frac{1}{4}} - \cdot0388 \right\}^2$$

$$= \left\{ \left(225 \times \frac{18\cdot42}{2}\sqrt{\cdot00005}\right)^{\frac{1}{4}} - \cdot0388 \right\}^2$$

$$= \left\{ \left(225 \times 9\cdot21 \times \cdot007\right)^{\frac{1}{4}} - \cdot0388 \right\}^2$$

$$= \left(\sqrt[4]{14\cdot5} - \cdot0388 \right)^2 = (1\cdot95 - \cdot0388)^2$$

$$= \overline{1\cdot91})^2 = 3\cdot648 \text{ feet per second.}$$

6.—Hagen's Formula.

Mean velocity—

$$v = 4{\cdot}39\sqrt{m}\cdot\sqrt[6]{i}$$
$$= 4{\cdot}39\sqrt{18{\cdot}42}\cdot\sqrt[6]{{\cdot}00005}$$
$$= 4{\cdot}39 \times 4{\cdot}29 \times {\cdot}191$$
$$= 3{\cdot}597 \text{ feet per second.}$$

7.—de Prony's with Wiesbach's Coefficients.

Mean velocity—

$$v = ({\cdot}00024 + 8675\,mi)^{\frac{1}{2}} - {\cdot}015$$
$$= ({\cdot}00024 + 9675 \times 18{\cdot}42 \times {\cdot}00005)^{\frac{1}{2}} - {\cdot}015$$
$$= ({\cdot}00024 + 7{\cdot}98968)^{\frac{1}{2}} - {\cdot}015$$
$$= \sqrt{7{\cdot}98992} - {\cdot}015$$
$$= 2{\cdot}827 - {\cdot}015$$
$$= 2{\cdot}812 \text{ feet per second.}$$

8.—Rankine's Formula.

Mean velocity—

$$v = 8{\cdot}025\sqrt{\frac{i\,m}{{\cdot}007565}} = \sqrt{8512\,im} = 92{\cdot}26\sqrt{im}$$
$$= 92{\cdot}26\sqrt{18{\cdot}42 \times {\cdot}00005}$$
$$= 92{\cdot}26\sqrt{{\cdot}000921}$$
$$= 92{\cdot}26 \times {\cdot}0303$$
$$= 2{\cdot}796 \text{ feet per second.}$$

Mean velocities.

(1)	By Kutter's formula	$v = 4{\cdot}445$	feet per second.
(2)	" de Prony's "	$v = 2{\cdot}879$	"
(3)	" Dupuit's "	$v = 2{\cdot}83$	"
(4)	" Chezy's "	$v = 3{\cdot}03$	"
(5)	" Humphrey's and Abbot's formula	$v = 3{\cdot}648$	"
(6)	" Hagen's formula	$v = 3{\cdot}597$	"
(7)	" de Prony's with Wiesbach's coefficients	$v = 2{\cdot}812$	"
(8)	" Rankine's formula	$v = 2{\cdot}796$	"
		8)26·047	
	Grand mean velocity	$v = 3{\cdot}256$	"

To find the probable errors by the method of least squares, assuming each of the formulas to have equal weights.

Number of Formula.	Velocity.	Difference from Mean.	Square of Residuals.
1	4·455	− 1·199	1·4376
2	2·879	+ 0·377	0·1421
3	2·83	+ 0·426	0·1815
4	3·03	+ 0·226	0·0511
5	3·648	− 0·392	0·1537
6	3·597	− 0·341	0·1163
7	2·812	+ 0·444	0·1971
8	2·796	+ 0·460	0·2116
Mean =	3·256		$[n\,n_1]$ = 2·4910

Probable errors—

$$r = {\cdot}6745\sqrt{\frac{[n\,n_1]}{m-1}} = {\cdot}6745\sqrt{\frac{2{\cdot}4910}{8-1}}$$
$$= {\cdot}6745\sqrt{0{\cdot}3560} = {\cdot}6745 \times \pm 0{\cdot}59$$
$$= \pm 0{\cdot}398 \text{ feet.}$$

Probable error of Arithemetical Mean—

$$r\,(x) = \frac{\cdot 6745}{\sqrt{m}}\sqrt{\frac{[n\,n_1]}{m-1}} = \frac{\cdot 6745}{\sqrt{8}}\sqrt{\frac{2\cdot 4910}{8-1}}$$
$$= \frac{\cdot 6745}{2\cdot 8284}\sqrt{0\cdot 3560} = \cdot 2385 \times \pm 0\cdot 59$$
$$= \pm\ 0\cdot 14 \text{ feet.}$$

Mean discharge..... $D = 3824\cdot 75 \times 3\cdot 256 = 12{,}460$ cubic feet per second.
" D in earth channel with water 22 feet deep = 12,448 "

Hence, the contracted channel in rock, with a head of 23·75 feet, discharges the same quantity as the larger channel in earth with a depth of 22 feet. In other words, the contraction causes an elevation of the water surface of 1·75 feet at the point of contraction, or the junction of the earth and rock channels.

1st.—CHANNEL IN EARTH.

The elevation of mean lake level of Lake Michigan over the monthly mean level of November, 1895, has been 2·6 feet

To find the discharge of the Chicago Channel at that elevation.

CROSS-SECTION.

Width at bottom..... 202 feet
Depth ..... 24·6 "
Side slopes—2 horizontal to 1 vertical—
Width at surface..... $(101 + 49\cdot 2) \times 2 = 300\cdot 4$ "
Sectional Area..... $A = \frac{202 + 300\cdot 4}{2} \times 24\cdot 6 = 6179\cdot 52$ square "
Wetted perimeter ..... $(101 + \sqrt{24\cdot 6^2 + 49.2^2}) \times 2$
$= (101 + 55) \times 2$
$= 156 \times 2 = 312$ feet.
Hydraulic mean radius..... $m = \frac{6179\cdot 52}{312} = 19\cdot 8$ feet.
Slope of channel..... $i = \frac{1}{40000} = \cdot 00025$ feet per foot linear.

1.—*Kutter's Formula.*

Mean velocity—

$$v = \left\{\frac{41\cdot 6 + \frac{1\cdot 811}{n} + \frac{\cdot 00281}{i}}{1 + \left(41\cdot 6 + \frac{\cdot 00281}{i}\right)\frac{n}{\sqrt{m}}}\right\}\sqrt{m\,i}$$
$$= \left\{\frac{41\cdot 6 + \frac{1\cdot 811}{\cdot 0275} + \frac{\cdot 00281}{\cdot 000085}}{1 + \left(41\cdot 6 + \frac{\cdot 00281}{\cdot 000025}\right)\frac{\cdot 0275}{\sqrt{19\cdot 8}}}\right\}\sqrt{19.8 \times 000025}$$
$$= \left\{\frac{41\cdot 6 + 65\cdot 85 + 112\cdot 4}{1 + \left(41\cdot 6 + 112\cdot 4\right)\frac{\cdot 0275}{4\cdot 45}}\right\}\sqrt{\cdot 090495}$$
$$= \left(\frac{219\cdot 85}{1 + 154 + \frac{\cdot 0275}{4\cdot 45}}\right) \times \cdot 02225$$
$$= \left(\frac{219.85}{1+154+\cdot 00618}\right) \times \cdot 02225$$
$$= \frac{219\cdot 85 \times \cdot 02225}{1 + \cdot 954}$$
$$= \frac{4\cdot 89}{1\cdot 954} = 2\cdot 5 \text{ feet per second.}$$

2.—de Prony's Formula.

Mean velocity—

$$\begin{aligned} v &= \left(\cdot 0237 + 9966\, m\, i\right)^{\frac{1}{2}} - \cdot 1542 \\ &= \left(\cdot 0237 + 9966 \times 19\cdot 8 \times \cdot 000025\right)^{\frac{1}{2}} - \cdot 1542 \\ &= \left(\cdot 0237 + 9966 \times \cdot 000495\right)^{\frac{1}{2}} - \cdot 1542 \\ &= \left(\cdot 0237 \times 4\cdot 9332\right)^{\frac{1}{2}} - \cdot 1542 \\ &= 2\cdot 227 - \cdot 1542 \\ &= 2\cdot 073 \text{ feet per second.} \end{aligned}$$

3.—Dupuit's Formula.

Mean velocity—

$$\begin{aligned} \cdot 0000323\, v^2 &= i(1 + \cdot 225\, m) \\ &= (1 + \cdot 225 \times 19\cdot 8) \times 000025 \\ &= (1 + 4\cdot 455) \times \cdot 000025 \\ &= \cdot 0001364 \end{aligned}$$

$$\therefore\ v^2 = \frac{\cdot 0001364}{\cdot 0000323} = 4\cdot 223$$

$$v = \sqrt{4\cdot 223} = 2\cdot 025 \text{ feet per second.}$$

4.—Chezy's Formula.

Mean velocity—

$$\begin{aligned} v &= c\sqrt{m\, i} \\ &= 100\sqrt{19\cdot 8 \times \cdot 000025} \\ &= 100\sqrt{\cdot 000495} \\ &= 100 \times \cdot 02225 \\ &= 2\cdot 225 \text{ feet per second.} \end{aligned}$$

5.—Humphreys and Abbot's Formula.

Mean velocity—

$$\begin{aligned} v &= \left\{\left(225\frac{m}{2}\sqrt{i}\right)^{\frac{1}{2}} - \cdot 0388\right\}^2 \\ &= \left\{\left(225 \times \frac{19\cdot 8}{2}\sqrt{\cdot 000025}\right)^{\frac{1}{2}} - \cdot 0388\right\}^2 \\ &= \left\{\left(225 \times 9\cdot 9 \times \cdot 005\right)^{\frac{1}{2}} - \cdot 0388\right\}^2 \\ &= \left(\sqrt[4]{11\cdot 137} - \cdot 0388\right)^2 = \left(1\cdot 827 - \cdot 0388\right)^2 \\ &= \overline{1\cdot 788}\Big)^2 = 3\cdot 19 \text{ feet per second.} \end{aligned}$$

6.—Hagen's Formula.

Mean velocity—

$$\begin{aligned} v &= 4\cdot 39\ \sqrt{m}\cdot\sqrt[6]{i} \\ &= 4\cdot 39\ \sqrt{19\cdot 8}\cdot\sqrt[6]{\cdot 000025} \\ &= 4\cdot 39 \times 4\cdot 45 \times \cdot 171 \\ &= 3\cdot 341 \text{ feet per second.} \end{aligned}$$

7.—de Prony's with Wiesbach's Coefficients.

Mean velocity—

$$v = (\cdot 00024 + 9675\, mi)^{\frac{1}{2}} - \cdot 013$$
$$= (\cdot 00024 + 8676 \times 19\cdot 9 \times \cdot 000025) - \cdot 015$$
$$= (\cdot 00024 + 4\cdot 29413)^{\frac{1}{2}} - \cdot 015$$
$$= \sqrt{4\cdot 29437} - \cdot 015$$
$$= 2\cdot 072 - \cdot 015$$
$$= 2\cdot 057 \text{ feet per second.}$$

8.—Rankine's Formula.

Mean velocity—

$$v = 8\cdot 025 \sqrt{\frac{m\, i}{\cdot 007565}} = \sqrt{8512\, mi} = 92\cdot 26 \sqrt{m\, i}$$
$$= 92\cdot 26 \sqrt{18\cdot 8 \times \cdot 000025}$$
$$= 92\cdot 26 \times \cdot 02225$$
$$= 2\cdot 053 \text{ feet per second.}$$

Mean velocities.

1.	By Kutter's formula	$v = 2\cdot 5$	feet per second.
2.	" de Prony's "	$v = 2\cdot 073$	"
3.	" Dupuit's "	$v = 2\cdot 055$	"
4.	" Chezy's "	$v = 2\cdot 225$	"
5.	" Humphrey's and Abbot's formula	$v = 3\cdot 197$	"
6.	" Hagen's formula	$v = 3\cdot 341$	"
7.	" de Prony's with Wiesbach's Coefficients	$v = 2\cdot 057$	"
8.	" Rankine's formula	$v = 2\cdot 053$	"
		$8\overline{)19\cdot 501}$	
	Grand mean velocity	$v = 2\cdot 438$	"

To find the probable errors by the method of least squares, assuming each of the formulæ to have equal weights :

No. of Formula.	Velocity.	Difference from Mean.	Square of Residuals.
1	2·5	− 0·062	0·0038
2	2·073	+ 0·365	0·1322
3	2·055	+ 0·383	0·1467
4	2·225	+ 0·213	0·0454
5	3·197	− 0·759	0·5761
6	3·341	− 0·901	0·8118
7	2·057	+ 0·381	0·1452
8	2·053	+ 0·385	0·1482
	Mean = 2·438		$[n\, n,] = 2\cdot 0104$

Probable error—

$$r = \cdot 6745 \sqrt{\frac{[n\, n,]}{m-1}} = \cdot 6745 \sqrt{\frac{2\cdot 0104}{8-1}}$$
$$= \cdot 6745 \sqrt{0\cdot 2872} = 6\cdot 745 \times \pm 0\cdot 53$$
$$= \pm 0\cdot 36 \text{ feet.}$$

Probable error of Arithmetical Mean—

$$r\,(x) = \frac{\cdot 6745}{\sqrt{m}} \sqrt{\frac{[n\, n,]}{m-1}} = \frac{\cdot 6475}{\sqrt{8}} \sqrt{\frac{2\cdot 0104}{8-1}}$$
$$= \frac{\cdot 6745}{2\cdot 8284} \sqrt{0\cdot 2872} = \cdot 2385 \times \pm 0\cdot 53$$
$$= \pm 0\cdot 126 \text{ feet}$$

Mean discharge.... .....$D = 6179\cdot 52 \times 2\cdot 438 = 15{,}066$ cubic feet per second.

RECAPITULATION.

Discharge of Chicago Drainage Channel with a

Depth of 22 feet	12,448	cubic feet	per second.
" 23 "	13,460	"	"
" 24 "	14,400	"	"
" 24·6 "	15,066	"	"
" 25 "	15,480	"	"
" 26 "	16,560	"	"
" 27 "	17,540	"	"

THE ELEVATION OF BACKWATER CAUSED BY CHANNEL CONTRACTION.

Let

Y = Elevation of water surface immediately behind the contracted channel. In this case = 1·75 feet.

y = Elevation at any other point up stream.

i = Slope of bottom of uncontracted channel or pond.

s = Distance up stream from the point of contraction. In this case at Robey Street junction = 70,000 feet.

1.—Guilhelm's Formula.

$$y = Y \sqrt[3]{\frac{1}{1 + \frac{1}{8} Y \left(\frac{i\,s}{Y}\right)^6} + \left(\frac{i\,s}{Y}\right)^3} - i\,s$$

$$= 1{\cdot}75 \sqrt[3]{\frac{1}{1 + \frac{1}{8} \times 1{\cdot}75 \left(\frac{{\cdot}000025 \times 70{,}000}{1{\cdot}75}\right)^6} + \left(\frac{{\cdot}000025 \times 70{,}000}{1{\cdot}75}\right)^3} - {\cdot}000025 \times 70{,}000$$

$$= 1{\cdot}75 \sqrt[3]{\frac{1}{1 + 35 \times 1} + 1} - 1{\cdot}75$$

$$= 1{\cdot}75 \sqrt[3]{\frac{1}{1{\cdot}35} + 1} - 1{\cdot}75 = 1{\cdot}75 \sqrt[3]{{\cdot}74 + 1} - 1{\cdot}75$$

$$= 1{\cdot}75 \sqrt[3]{1{\cdot}74} - 1{\cdot}75$$

Log. 1·74 = 0·2405492 ÷ 3 = 0·0801831 = 1·2

$\therefore y = 1{\cdot}75 \times 1{\cdot}2 - 1{\cdot}75 = 2{\cdot}1 - 1{\cdot}75 = 0{\cdot}35$ feet. Height of backwater at Robey Street Junction.

2.—Funk's Formula.

$$y = 2\,Y - \left\{ i\,s + \sqrt{Y\left(Y - \tfrac{1}{2} i\,s\right)} \right\}$$

$$= 2 \times 1{\cdot}75 - \left\{ {\cdot}000025 \times 70{,}000 + \sqrt{1{\cdot}75 \left(1{\cdot}75 - \tfrac{1}{2} {\cdot}000025 \times 70{,}000\right)} \right\}$$

$$= 3{\cdot}5 - \left\{1{\cdot}75 + \sqrt{1{\cdot}75\,(1{\cdot}75 - 5 \times 1{\cdot}75}\right\}$$

$$= 3{\cdot}5 - \left\{1{\cdot}75 + \sqrt{1{\cdot}75\,(1{\cdot}75 - {\cdot}875}\right\}$$

$$= 3{\cdot}5 - \left\{1{\cdot}75 + \sqrt{1{\cdot}75 + {\cdot}875}\right\}$$

$$= 3{\cdot}5 - \left\{1{\cdot}75 + \sqrt{1{\cdot}53125}\right\}$$

$$= 3{\cdot}5 - (1{\cdot}75 + 1{\cdot}25)$$

$$= 3{\cdot}5 - 3$$

$$= 0{\cdot}5 \text{ feet.}$$ Elevation of backwater at Robey St. junction.

3.—Poiree's Formula.

$$y = Y - is + \sqrt{\frac{is}{4\,Y}}\Big)^2$$

$$= 1{\cdot}75 - 1{\cdot}75 + \sqrt{\frac{1{\cdot}75}{4 + 1{\cdot}75}}\Big)^2$$

$$= 1{\cdot}75 - 1{\cdot}75 + \frac{3{\cdot}0625}{7}$$

$$= {\cdot}4375 \text{ feet.}$$ Height of backwater at Robey Street junction.

4—Dupuit's Formula.

$\text{Log } y = \log Y - \frac{i\,s}{{\cdot}77\,P}$; in which P is the mean depth between the points.

$$\text{Log } y = \log 1{\cdot}75 - \frac{{\cdot}000025 \times 70000}{{\cdot}77 \times 23}$$

Reduced to inches gives—

$$\text{Log } 12\,y = \log 21 - \frac{1{\cdot}75}{1{\cdot}77}$$

$$= 1{\cdot}3222193 - 1 = 0{\cdot}3222193$$

$$\therefore \quad y = \frac{0{\cdot}3222193}{12} = 0{\cdot}027 \text{ feet.}$$ Height of backwater at Robey Street junction.

(1) By Guilhelm's Formula	$y = 0{\cdot}35$	feet.
(2) By Funk's "	$y = 0{\cdot}5$	"
(3) By Poiree's "	$y = 0{\cdot}437$	"
(4) By Dupuit's "	$y = 0{\cdot}027$	"
	4)1·314	
Mean height	$y = 0{\cdot}3285$	"

To find the probable errors by the method of least squares, assuming each of the formulas to have equal weights.

No. of Formula.	Depth of Backwater.	Difference from Mean.	Square of Residuals.
1	0·35	—0·021	0·0004
2	0·5	—0·171	0·0292
3	0·437	—0·108	0·0117
4	0·027	+0·301	0·0906
Mean =	0·3285	$[nn_1]$ =	0·1319

Probable error :—

$$r = {\cdot}6745\sqrt{\frac{[n\ n_1]}{m-1}} = {\cdot}6745\ \sqrt{\frac{0{\cdot}1319}{4-1}}$$

$$= 6745\ \sqrt{0{\cdot}044} = {\cdot}6745 \times \pm\ 0{\cdot}21$$

$$= \pm\ 0\ 14 \text{ feet.}$$

Probable error of Arithmetical Mean :—

$$r\,(x) = \frac{{\cdot}6745}{\sqrt{m}}\sqrt{\frac{[n\ n_1]}{m-1}} = \frac{{\cdot}6745}{\sqrt{4}}\sqrt{\frac{0{\cdot}1319}{4-1}}$$

$$= \frac{{\cdot}6745}{2}\sqrt{0{\cdot}044} = {\cdot}3372 \times \pm\ 021$$

$$= \div\ 0{\cdot}071 \text{ feet.}$$

TANK ILLUSTRATIONS.

Suppose a tank 5 feet long, and 2 feet wide, with a depth of water of 10 feet, receives a constant supply of 3 cubic feet per second, what will be the diameter of a round orifice in the bottom capable, with a constant head of 10 feet, to discharge 3 cubic feet per second ?

Let D = Discharge in cubic feet per second.
h = Depth of water or constant head.
A = Area of orifice in square feet.
c = Co-efficient of discharge.

By Rankine's Formula.

$$D = 8{\cdot}025\, c\, A \sqrt{h}$$
$$= 8{\cdot}025 \times {\cdot}618\, A \sqrt{10}$$
$$= 8{\cdot}025 \times {\cdot}618 \times 3{\cdot}16\, A$$
$$= 15{\cdot}67\, A$$
$$\therefore \quad A = \frac{D}{15{\cdot}67} = \frac{3}{15{\cdot}67}$$
$$= {\cdot}1915 \text{ square feet.}$$

Let x = diameter of orifice in feet.

Then—

$${\cdot}7854\, x^2 = {\cdot}1915$$
$$\therefore \quad x^2 = \frac{{\cdot}1915}{{\cdot}7854} = {\cdot}2439$$
$$x = \sqrt{{\cdot}2439} = {\cdot}494 \text{ feet.}$$
$$= 5{\cdot}84 \text{ inches.}$$

2ndly. Suppose now a second orifice, made in the bottom of the tank, capable of discharging ·2 cubic feet per second at a like head of pressure; what would be the diameter of the orifice?

Let the same formula and notation be used.

A' = area of second orifice.

Then—

$$A' = \frac{D}{15{\cdot}67} = \frac{{\cdot}2}{15{\cdot}67} = {\cdot}01277 \text{ square feet.}$$

Let x' = diameter of less orifice.

Then—

$${\cdot}7854\, x'^2 = 01277$$
$$\therefore \quad x'^2 = \frac{{\cdot}01277}{{\cdot}7854} = {\cdot}0164$$
$$x' = \sqrt{{\cdot}0164} = {\cdot}128 \text{ feet} = 1{\cdot}54 \text{ inches.}$$

Suppose now, with a constant supply of three cubic feet, both orifices are opened, discharging 3·2 cubic feet per second, the problem is to find how much the level of the tank must fall to establish equilibrium between influx and efflux.

At first blush, one would be led to infer that, with the outflow in excess of the inflow, the tank would eventually be drained of all its contents. But this is not so, as the following computation demonstrates:

Let h_1 = equal the depth of water in the tank or new head of pressure to maintain equilibrium of supply and disdharge.

Then

$$D = 8{\cdot}025\, c\, (A + A_1) \sqrt{h_1}$$
$$= 8{\cdot}025 \times {\cdot}618\, ({\cdot}1915 + {\cdot}01277) \sqrt{h_1}$$
$$= 8{\cdot}025 \times {\cdot}618 \times {\cdot}2043 \sqrt{h_1}$$
$$= 1{\cdot}0133 \sqrt{h_1}$$

Square both sides.

Then

$$D^2 = \overline{1{\cdot}0133})^2\, h_1$$
$$h_1 = \left(\frac{1{\cdot}0133}{D}\right)^2 = \left(\frac{3}{1{\cdot}0133}\right)^2$$
$$= \overline{2{\cdot}96})^2 = 8{\cdot}76 \text{ feet.}$$

∴ to maintain equilibrium, there would be a depression in the water surface of the tank of 1·24 feet; and it would remain at this elevation whiie the conditions of the problem remained unchanged.

Suppose, now, that we plug up both orifices in the bottom of the tank, and substitute an orifice or outlet at the surface, and, for simplicity of calculation, call it a rectangular notch extending upward to the top of the tank. Let the other conditions of the problem remain the same, viz., a constant influx and efflux of three cubic feet per second.

Assuming the notch to be two feet in width, what will be its depth from the surface of the water to the sill to discharge 3 cubic feet per second?

Let b = the width, while the rest of the notation remains the same as above.

By Rankine's Formula, the discharge

$$D = 8{\cdot}025\, c \times \tfrac{2}{3}\, b\, h_1^{\frac{3}{2}}$$

$$= 5{\cdot}35 \times {\cdot}5 \times 2\, h_1^{\frac{3}{2}}$$

$$= 5{\cdot}35\, h_1^{\frac{3}{2}}$$

$$\therefore h_1^{\frac{3}{2}} = \frac{D}{5{\cdot}35}$$

$$= \frac{3}{5{\cdot}35} = {\cdot}56.$$

Squaring both sides.

$$h_1^{3} = {\cdot}56^2$$

$$h_1 = {\cdot}56^{\frac{2}{3}}$$

$$\log.\ {\cdot}56 = \overline{1}\ {\cdot}7481880 \times \tfrac{2}{3}$$

$$= \overline{1}\ {\cdot}8221353.$$

$${\cdot}68 \text{ feet} = 8{\cdot}16 \text{ inches.}$$

Again, suppose that a second rectangular notch is made in the tank capable of discharging ·2 cubic feet per second, with the sill of both orifices or notches at the same elevation; what will be the width, b_1, of the second orifice?

Using the same notation and formula—

$$D = 8{\cdot}025\, c \times \tfrac{2}{3}\, b_1 h_1^{\frac{3}{2}}$$

$$= 5{\cdot}35 \times {\cdot}5 \times {\cdot}56\, b_1$$

$$= 1{\cdot}498\, b_1$$

$$\therefore b_1 = \frac{D}{1{\cdot}498} = \frac{{\cdot}2}{1{\cdot}498}$$

$$= {\cdot}1335 \text{ feet} = 1{\cdot}6 \text{ inches.}$$

Next to find the height of the surface of the water in the tank above the sill of the notch, when the two orifices or notches will discharge 3 cubic feet per second, that is to say, when the equilibrium between influx and efflux is again restored?

Let x be the difference of elevation between the original depth of water on the sill when the one notch is discharging 3 cubic feet per second, and that when the two notches are discharging 3·2 cubic feet per second.

Then $h_1 - x$ will be the depth of water on the sill when equilibrium is again established between inflow and outflow.

Call b_1 = combined width of both notches, the other notation remaining the same. Then

$$D = 8{\cdot}025\, c \times \tfrac{3}{2}\, b_1 h_1^{\frac{3}{2}}$$

$$= 5{\cdot}35 \times {\cdot}5 \times 2{\cdot}1335\, h_1^{\frac{3}{2}}$$

$$= 5{\cdot}707\, h_1^{\frac{3}{2}}$$

$$\therefore h_1^{\frac{3}{2}} = \frac{D}{5{\cdot}707}$$

$$= \frac{3}{5{\cdot}707}$$

$$= {\cdot}52567$$

Squaring both sides—

$$h_1^3 = \overline{\cdot 52567}\Big)^2$$
$$h_1 = \overline{\cdot 52567}\Big)^{\frac{2}{3}}$$
$$\log. \cdot 52567 = \overline{1}\cdot 7207132 \times \tfrac{2}{3}$$
$$\cdot 65 = \overline{1}\cdot 8138088$$
$$\text{Hence, } x = \cdot 68 - \cdot 65 = \cdot 03 \text{ feet}$$
$$= \cdot 36 \text{ inches.}$$

This is the depression in the water surface of the tank necessary to maintain equilibrium between influx and efflux.

When the sills of the notches are at different elevations, the problem becomes more complicated, involving a cubic equation for the value of x. While the roots of the equation could be calculated by "Taylor's Theorem," it has not been deemed necessary for purposes of this illustration to pursue it any further.

In the above illustrations, it will be observed that the discharge of the smaller notch or orifice is 1 : 15 of the capacity of the larger. This ratio has been assumed as that of the Chicago drainage channel to that of Lake Huron-Michigan by its natural outlet, the St. Clair River.

LEVELS OF THE GREAT LAKES.

The plane of reference of the water level curves of Lake Erie, adopted by the Lake Survey, is the supposed high water of 1838. It is 575·2 feet above mean tide in New York Harbour. With this plane of reference as zero, mean level of surface of the lake from 1860 to 1875 is —2·34 feet. This latter, which is 572·86 feet above mean tide, New York, is the zero of the United States gauge at Buffalo Harbour. The mean monthly level of Lake Erie for the month of November, 1895, at Cleveland, Ohio, has been —4·41 feet, equivalent to —2·07 feet on the Buffalo gauge.

For the discharge measurements of the Niagara River, taken in December, 1891, and in April and May, 1892, (see annual report of the Chief of Engineers, U. S. Army, for 1893, p. 4364 *et sequitur*), the relation of the Buffalo gauge to the local gauge at the discharge cross-section is expressed by the following equation :—

Local gauge height $= 2\cdot087 + 0\cdot624\,x - 0\cdot046\,x^2 +$ &c., in which $x =$ height of Buffalo gauge in feet, — + indicating above zero, and — below.

$$-2\cdot07 \text{ feet Buffalo gauge} = 2\cdot087 + 0\cdot624 \times (-2\cdot07) - 0\cdot046 \times \overline{-2\cdot07}\Big)^2 + \text{&c.}$$
$$= 2\cdot087 - 1\cdot292 - 0\ 197 = 0\cdot598$$

The discharge given on the smooth curve of discharge for 0·6 feet on local gauge is 190,000 cubic feet per second.

Assuming that 85 per cent of this efflux passed through the St. Clair River at the foot of Lake Huron, gives $190{,}000 \times \cdot 85 = 161\cdot500$ cubic feet per second, the discharge for Lake Huron. At a corresponding stage of Lake Michigan, the Chicago drainage channel will discharge, as shown above (p. 13) 12,500 cubic feet per second.

Assuming like conditions as in our tank illustrations, that is to say, a rectangular outlet at Port Huron, 2,000 feet in width.

1st. What would be the depth (mean) to dischaoge 161,500 cubic feet per second?

By Rankine's formula above—

$$D = 8\cdot025c \times \tfrac{2}{3}\, b\, h_1^{\frac{3}{2}}$$
$$= 5\cdot35 \times \cdot 5 \times 2000\, h_1^{\frac{3}{2}}$$
$$= 5350\, h_1^{\frac{3}{2}}$$
$$\therefore \quad h_1^{\frac{3}{2}} = \frac{D}{5350} = \frac{161500}{5350} = 30\cdot2$$

Hence, squaring both sides,

$$h_1^3 = \overline{30\cdot2}\,)^2 \text{; and } h_1 = 30\cdot2^{\frac{2}{3}}$$
$$\log. 30\cdot2 = 1\cdot4800069 \times \tfrac{2}{3} = 0\cdot9866713,$$

and the natural number corresponding to this log. is 9·6978 feet, or 9·7 feet nearly.

2nd. What will be t' e depth when the discharge is reduced by the Chicago drainage channel to 161,500 – 12,500 = 149,000 cubic feet per second ?

As above, with same notation—

$$h_1^{\frac{3}{2}} = \frac{D}{5350} = \frac{149000}{5350} = 27{\cdot}85$$

Square both sides. Then

$$h_1^{\ 3} = 27{\cdot}85^2 \text{ ; and } h_1 = 27{\cdot}85^{\frac{2}{3}}$$

$$\log.\ 27{\cdot}85 = 1{\cdot}4448252 \times \tfrac{2}{3}$$

$$9{\cdot}2943 = 0{\cdot}9632168$$

the natural number to this log.

Height or head of water on the outlet or submerged dam at foot of Lake Huron when discharging 161,500 cubic feet per second.....	9·6978	feet.
Height or head of water on the outlet or submerged dam at foot of Lake Huron when discharging 149,000 cubic feet per second.....	9·2043	"
	0·4035	"
	12	
Depresssion =	4·842	inches.

By the smooth curve of discharge of the Niagara River measurements, the discharge at mean level at Lake Erie, is 232,800 cubic feet per second. Assuming that 85 per cent is discharged by Lake Huron through the St. Clair river, is equal to 198,000 cubic feet per second.

Using the same formula and notation as above—

$$D = 8{\cdot}025\, c \times \tfrac{2}{3}\, b\, h_1^{\frac{3}{2}}$$

$$= 5{\cdot}35 \times {\cdot}5 \times 2000\, h_1^{\frac{3}{2}} = 5350\ h_1^{\frac{3}{2}}$$

$$\therefore\ h_1^{\frac{3}{2}} = \frac{D}{5350} = \frac{198{,}000}{5350} = 37$$

$$h_1^{\ 3} = 37^2 \text{ ; and } h_1 = 37^{\frac{2}{3}}$$

$$\log.\ 37 = 1{\cdot}5682017 \times \tfrac{2}{3}$$

$$11{\cdot}1 = 1{\cdot}0454678.$$

At this stage, the Chicago drainage channel, as shown at p. 38 will discharge 15,066 cubic feet per second, diminishing to that extent the discharge of Lake Huron.

This is 198,000 - 15,000 = 183,000 cubic feet per second.

By the same process as above—

$$h_1^{\frac{3}{2}} = \frac{183{\cdot}000}{5{\cdot}350} = 34{\cdot}2$$

$$h_1^{\ 3} = 34{\cdot}2)^2 \text{ ; and } h_1 = 34{\cdot}2^{\frac{2}{3}}$$

$$\log.\ 34{\cdot}2 = 1{\cdot}5340261 \times \tfrac{2}{3}$$

$$10{\cdot}563 = 1{\cdot}0226841$$

the corresponding natural number.

Height or head of wate on outlet at foot of Lake Huron, discharging, at mean stage, 198,000 cubic feet per second.	11·1	feet.
Height or head of water on outlet at foot of Lake Huron, discharging 183,000 cubic feet per second, the efflux as reduced by the Chicago drainage channel.....	10·563	"
	0·537	"
	12	
Depression of surface.....	6·444	inches.

When the Buffalo gauge registers + 1·8 feet, which is – 0·54 feet below the plane of reference of Lake Erie, the smooth curve of discharge of the Niagara River measurements indicates a discharge of 263,000 cubic feet per second. This elevation corresponds closely to an elevation of Lake Michigan of five feet above the monthly mean level of November, 1895.

Taking 85 per cent of this discharge as the outflow of Lake Huron in its normal condition, it becomes 263,000 × ·85 = 223,550 cubic feet per second.

The discharge by the Chicago drainage channel at this elevation as computed above (p. 33), is 17,540 cubic feet per second, diminishing in its abnormal condition the efflux of Lake Huron.

This gives 223,550 − 17,540 = 206,000 cubic feet per second.

As above

$$D = 8\cdot025c \times \tfrac{2}{3}bh_1^{\frac{3}{2}} = 5\cdot35 \times \cdot5 \times 2000h_1^{\frac{3}{2}}$$
$$= 5350h_1^{\frac{3}{2}}$$
$$\therefore h_1^{\frac{3}{2}} = \frac{D}{5350} = \frac{223,550}{5350} = 41\cdot785$$

Squaring both sides—

$$h_1^3 = 41\cdot785)^2 \text{ ; and } h_1 = 41\cdot785.5^{\frac{2}{3}}$$
$$\log.\ 41\cdot785 = 1\cdot6200204 \times \tfrac{2}{3}$$
$$12\cdot023 = 1\cdot0800136$$

the natural number corresponding to this log.

2nd. What will be the depth when the efflux is reduced to 206,000 cubic feet per second.

In the same manner as above—

$$h_1^{\frac{3}{2}} = \frac{D}{5350} = \frac{206,000}{5350} = 38\cdot5$$
$$\therefore h_1^3 = 38\cdot5^2 \text{ ; and } h_1 = 38\cdot5^{\frac{2}{3}}$$
$$\log.\ 38\cdot5 = 1\cdot5854607 \times \tfrac{2}{3}$$
$$11\cdot402 = 1\cdot0569738$$

the corresponding natural number.

Height or head of water on outlet at foot of Lake Huron, discharging, at high stage, 223,550 cubic feet per second	12·023 feet.
Height or head of water on outlet at foot of Lake Huron, discharging 206,000 cubic feet per second, the outflow as reduced by the Chicago drainage channel	11·402 "
	0·621 "
	12
Depression of lake surface	7·452 inches.

RECAPITULATION.

Depression of lake surface at lowest stage	4·892 inches.
" " mean "	6·444 "
" " highest "	7·452 "

It is not probable that these quantities respectively will vary one inch either way. It is scarcely necessary to add that, of all the lakes, the maximum effect will be felt in Lake Huron-Michigan.

THE TIME TAKEN TO DEPRESS THE SURFACE OF LAKE.

The combined area of Lakes Huron-Michigan is 46,250 square miles. This is equal to 46,250 × 27,878,400 = 1,289,376,000,000 square feet. With a depth of one inch this area will contain 107,448,000,000 cubic feet of water.

Assuming a mean average abstraction of 15,000 cubic feet per second from Lake Michigan by the Chicago drainage channel, it will take 107,448,000,000 ÷ 15,000 = 7,163,200 seconds to lower the surface of Lake Huron-Michigan one inch.

7,163,200 ÷ 3,600 = 2,000 hours, nearly = 83 days.
For a depression of 3 inches gives 83 × 3 = 249 "

Assuming that, in a lowering of 3 inches, the surface area will be contracted about 1 : 1000 part of its original size, we have approximately a water area of 46,200 square miles. 46,200 square miles = 46,200 × 278,784,000 = 1,287,982,080,000 square feet.

With a depth of one inch, this area will contain 107,331,840,000 cubic feet of water. At a mean discharge per the Chicago drainage channel of 15,000 cubic feet per second, it will take $107,331,840,000 \div 15,000 = 7,155,456$ seconds to depress the surface of the lake one inch. $7,155,456 \div 3,600 = 1,988$ hours $= 82\frac{5}{6}$ days. And to depress the lake surface $3\frac{1}{2}$ inches will take $82\frac{5}{6} \times 3\frac{1}{2} = 290$ days.

To depress the lake surface the first	3 inches	.....	249 days.
" " " last	$3\frac{1}{2}$ "	.....	290 "
" " "	$6\frac{1}{2}$ "	.....	539 "

The foregoing calculations have not been verified.

EXTINCT OUTLET OF LAKE MICHIGAN.

The evidence in support of the theory that Lake Michigan, at some remote period of its history, discharged a part, at least, of its waters into the Gulf of Mexico by way of the Desplaines, Illinois and Mississippi rivers, is not only convincing and conclusive, but overwhelming.

THE HARVEST.

A report is the fruition, the crop, of an inquiry of this character. There is practically little else to show for the labour expended. Hence, if there be no report, there is no fruit, the result seems barren, the energy is wasted. When, as in this case, the report is only partial, the result may seem wholly incommensurate with the expenditure, a very small mouse indeed. But to judge fairly, it should be borne in mind that the report itself is only a small part,—the tangible part indeed,—a mere fraction of the labour involved in the preparation, in the accumulation and sifting of material in the initial or preliminary stages of the investigation. Very frequently the labour and outlay between a full report and no report at all, that is a report nipped in the bud, differs very little, although in the latter predicament there is practically nothing to show for the expenditure.

THANKS.

It affords me much pleasure to avail myself of this opportunity to acknowledge my obligations and return my best thanks to the following gentlemen for favours and courtesies conferred on me in connection with this service:

To all the gentlemen of the staff of the Marine and Fisheries Department with whom I have had official connection, for the very great kindness on all occasions, and for the cheerfulness and promptness they invariably evinced in affording me information and assistance.

To Hon. Edward Murphy, jr., United States Senator for the State of New York, for his great kindness in procuring for me through the public departments at Washington, several valuable reports, which have been to me of exceedingly great use in this investigation.

To Hon. J. B. Riley, United States Consul General, Ottawa, for many favours, not the least of which have been books, reports, and public documents procured from Washington, very rare and valuable, and difficult to obtain. Words fail to convey my appreciation of his kindness.

To General William P. Craighill, Chief of Engineers, United States Army, for all the information and other favours conferred on me, and the promptness with which he responded to my inquiries, both directly and indirectly.

To Major W. L. Marshall, Corps of Engineers, United States Army, in charge of the United States Engineer Office, Chicago, for drawings and blue prints of the United States survey of the proposed canal connecting Lake Michigan with the Mississippi River.

To Captain Thomas W. Symons, Corps of Engineers, in charge of the United States Engineer Office, Buffalo, N.Y., for a typewritten copy with tracing, of the unpublished report of Mr. J. C. Quintus of the discharge measurements of the Niagara River made in December, 1891.

To Mr. J. C. Quintus for his kindness in forwarding my letter to Captain Symons.

To Captain George A Zinn, Corps of Engineers, United States Army, in charge of United States Engineer Office, Milwaukee, Wis., for his courtesy and kindness in replying to my letter, and inclosing a copy of the monthly means of the water levels of Lake Michigan, taken at Milwaukee, from the month of July, 1894, to November, 1895, both inclusive.

To Col. Jared A. Smith, Corps of Engineers, United States Army, in charge of United States Engineer Office, Cleveland, O., for his courtesy and kindness in promptly replying to my letter, and accompanying it with a copy of the monthly mean levels of Lake Erie, from the month of July, 1894, to November, 1895, recorded at Cleveland, Monroe, Mich., Ashtabula, O., and Erie, Pa.

To Major W. S. Stanton, Corps of Engineers, United States Army, in charge of the United States Engineer Office, Oswego, N.Y., for his courtesy and kindness in promptly replying to my communication, and inclosing therewith a copy of the monthly means of the water levels of Lake Ontario, from July, 1894, to November, 1895, both inclusive, taken at Oswego.

To Captain H. E. Waterman, Corps of Engineers, United States Army, Secretary of the Mississippi River Commission, St. Louis, for his courtesy in sending me several copies of the Annual Reports of the Mississippi River Commission, from its inception to 1894, with few exceptions.

To John A. Ockerson, Esq., United States Engineer, Mississippi River Commission, St. Louis, for his great kindness in having these volumes sent me through the secretary.

To Col. A. MacKenzie, Corps of Engineers, United States Army, Washington, D.C., for his prompt and courteous reply to my communication to the Chief of Engineers, a copy of which is found in Appendix B.

To Major E. H. Ruffner, Corps of Engineers, United States Army, Buffalo, N.Y., and now of Baltimore, Md., one of the Board of Engineers appointed by the United States Government to report on the effect of the Chicago drainage channel on the levels of the Great Lakes, for his prompt and courteous reply to my communication relative to the report of the board.

MISCARRIED.

I wrote Mr. Charles H. Keep, secretary of the Lake Carriers' Association, to Cleveland, O., for a copy of his estimate of the commercial losses by a reduction of the levels of the lakes, referred to in the report of the Board of Engineers. Not knowing his post office address, it is probable that my letter miscarried, as I have had no reply.

All of which is most respectfully submitted.

J. L. P. O'HANLY,
Civil Engineer.

OTTAWA, February 29, 1896.

LIST OF DRAWINGS TO ACCOMPANY REPORT.

Plate No. 1.—Monthly mean of the water level curves of Lake Huron-Michigan, from June, 1859 to November, 1895, inclusive.

Plate No. 2.—Monthly mean of the water level curves of Lake Erie, from June, 1859, to November, 1895, inclusive.

Plate No. 3.—A tracing of Major Marshall's Map of that part of the United States Survey of the proposed waterway connecting Lake Michigan with the Mississippi River, between Lake Michigan and La Salle, Ill.

Plate No. 4.—Tracing of Lake Erie, with profile of the storm of October 14, 1893.

Plate No. 5.—Cross-section of the Desplaines Valley at Lemont, showing the proposed Chicago Drainage Channel, the Illinois and Michigan Canal, the Desplaines and diverted Desplaines River, &c.

Plate No. 6—Map showing the great lakes and their drainage area or basin, with the Desplaines and Illinois Rivers, and the Mississippi.

Plate No. 7.—The velocity, discharge and gauge relation curves of the United States discharge measurements of the Niagara River in December, 1891, and in April and May, 1892.

Plate No. 8.—The discharge cross-sections of the Niagara River at Black Rock, and Lower Black Rock, at the foot of Lake Erie.

Plate No. 9.—Cross-sections, in earth and rock, of the Chicago Drainage Channel.

(Copy.)

APPENDIX A.

Report of the Board of Engineers, appointed by the United States, on the probable effect of the Chicago Drainage Channel on the levels of the great lakes, addressed to General William P. Craighill, Chief of Engineers, United States Army.

(From the New York *Engineering News* October 3, 1895.)

The board met in Chicago, August 12, 1895, and on August 13th and 14th accompanied the officers of the drainage canal over the line under construction. Every facility and courtesy possible has been extended by the trustees and engineers of the canal for a full investigation of the subject matter. A brief description of the canal is extracted from the printed report furnished the board by these gentlemen:

The main drainage channel of the sanitary district of Chicago is now under contract from its confluence with the south branch of the Chicago River, at Robey Street, in the city of Chicago, to its southern terminus, in Will County, Ill. At the southern end of the channel the controlling works will be located. Beyond these works, the construction contemplated by the district will be the work necessary for conducting the flow from the channel in conjunction with the waters of the Desplaines River, down the declivity to and through the city of Joliet, and making such change in the Illinois and Michigan Canal as the new conditions developed will make necessary.

The first work put under contract extended south-westerly from the Willows Springs Road, and these sections were numbered consecutively, Nos. 1 to 14.

Average length of sections, one mile. Easterly from Willow Springs road, the sections are lettered from A to O, omitting J. The lettered sections are, except for a short distance near Summit, entirely in glacial drift, defined in the specifications thus: "Glacial drift shall comprise the top soil, earth, muck, sand, gravel, clay, hardpan, boulders, fragmentary rock displaced from its original bed, and any other material that overlies the bedrock."

The sections from 1 to 14 were put under contract in July, 1892; from A to F were put under contract late in 1892 and early in 1893, and G to M inclusive were contracted for in December, 1893. Sections N and O were put under contract May 2, and Section 15, August 27, 1894. Earth was first broken on "Shovel Day," September 3, 1892, on the rock cut below Lemont.

The Desplaines Valley is traversed by the river from which it takes its name, a stream of wide fluctuations, with no constant and reliable fountain supply. During some seasons its whole discharge would pass through a 6-in. pipe, and at others its volume reaches 800,000 cubic feet per minute. Then it rolls majestically along, flooding the whole valley. Such being the situation, control of this stream was a condition precedent to the successful prosecution of the work upon the main channel. This control has been secured by the outlay of nearly $1,000,000 in constructing what is known as the River Diversion channel.

About 13 miles of new river channel had to be excavated with the location of the Main Drainage channel, and about 19 miles of levee built to divorce the waters of the Desplaines watershed from the channel which is to receive the waters of Lake Michigan, and pass them on to the Mississippi River, *viâ* the Lower Desplaines and Illinois Rivers. The width of the river diversion channel on the bottom is 200 feet, side slopes 1½ to 1, grade generally 0·12 feet per 1,000 feet.

At the head of this river diversion it was necessary to provide a safety valve in the form of a spillway, to allow surplus water to flow towards Chicago, because arrangements have not as yet been perfected for carrying the entire flood waters of the Desplaines through Joliet.

This spillway is a concrete dam capped with cut stone, and its wings faced with stone masonry. It is 397 feet long and its crest is 16·25 feet above Chicago datum

(this datum is referred to the low water of Lake Michigan of 1847, and is 579·61 feet above sea level at Sandy Hook). No water flows over this spillway until the volume passing the water-gage above it reaches 300,000 cubit feet per minute.

The cross section of the earth sections from A to E inclusive is 202 feet on the bottom, with side slopes of 2 to 1. This section extends for about 500 feet into the west end of F, and then reduces to 110 feet on the bottom, preserving the same side slopes. The explanation for this change of cross section is as follows:—Throughout the rock sections and those sections in whcih there is a preponderance of hard material, or where rock may appear, the section adopted is designed, according to law, for a flow of 600,000 cubic feet of water per minute, which means provision for a population of 3,000,000 people. The narrow channel provides for a flow of 300,000 cubic feet per minute, or for about the present population of Chicago. The enlargement of the narrow channel can be made by the easier methods of excavation, such as dredging, whenever the needs of the city require it. The grade throughout the lettered sections is 1 foot in 40,000 (·025 feet per 1,000 feet) and the bottom of the channel at Robey Street is 24·448 feet below datum. The numbered sections, from No. 1 to No. 6 inclusive, are underlaid with solid rock. The width of the bottom, in rock, is 160 feet, and walls of masonry laid in cement will be built upon the rock surface to a height of 5 feet above datum. Sections 7 to 14, inclusive, are in solid rock; width at bottom, 160 feet; sides vertical, prism taken out in three slopes with offsets of 6 inches on each side for each cut, making top width of 162 feet; grade in rock, 1 foot in 20,000 (·05 feet per 1,000.)

Section No. 15 is also in rock, and its cross section is enlarged at its south end so as to form a "windage basin," in which large vessels may be turned around. The controlling works are located on this section. These works will consist of gates or movable dams, by which the flow of water from the main channel into the tail race, which is to deliver the outflow into the Desplaines River can be controlled.

This river below Lockport follows the trough of the valley down a steep declivity to the canal basin in Joliet. The fluctuations in Lake Michigan by varying slope of water surface, will be felt at the controlling works, and provision must be made to meet these fluctuations within a range of 5 feet above datum, and 8 feet below, or an extreme oscillation of 13 feet. The fall from datum at the controlling works to the level of the upper basin will be about 42 feet in a distance of about 4⅛ miles. As the plans for controlling works have not been finally adopted by the Board of Trustees, they cannot now be discussed.

The total amount of excavation involved in the construction of the main channel is 26,077,765 cubic yards of glacial drift, and 12,071,668 cubic yards of solid rock, or an aggregate of 38,149,433 cubic yards, to which must be added the material excavated from the river diversion: glacial drift, 1,564,403 cubic yards; solid rock, 258,926 cubic yards; total river diversion, 1,823,329 cubic yards; grand total, main channel and river diversions, 39,972,762 cubic yards. All of this work is now under contract, and in addition thereto 384,958 cubic yards of retaining wall.

In response to the request of the senior member of the Board, the Board of Trustees of the Sanitary District of Chicago has furnished a report on lake level effects on account of the main channel of the Sanitary District of Chicago, containing briefs by Trustee L. E. Cooley, C.E., and by Thos. T. Johnston, Assistant Chief Engineer, accompanied by numerous blue prints. These papers present a full discussion of the subject as viewed by the canal officials.

What is the outflow af the Lower Lakes?

In November, 1891, the Chief of Engineers, U.S.A., at the request of the secretary of the American Society of Civil Engineers (who had been asked by the chief engineer of the Montreal Harbour Commission of Canada to suggest the subject), ordered a set of observations made to determine the amount of water flowing down the Niagara river. The time was especially propitious, as the water was then very low.

The results of these measurements were somewhat unexpected, and they were repeated in May, 1892. The second set corroborated the first, and the whole formed the

subject of a report to the Chief of Engineers, which appeared in his annual report of 1893, pages 4, 364 and following. But, as the subject was important, the *Engineering News* anticipated the appearance of the official report by publishing in its issue of March 2, 1893, this report, with the permission of the chief of engineers. This publication was the first ever made in which, as a result of careful measurements, a relation between the level of the lakes and their outflow, or discharge, had been established and given to the public. Prior determination of this discharge had not attempted to detect this relation, and nothing more than a general determination of a season's work had been published. In all plans for the Chicago Drainage Canal, the early measurements had been taken, and those studying the subject chose such isolated figures as suited them best.

The report of 1892, being so late in appearance, long after the drainage canal was put under construction, escaped the notice of many who are interested in navigation for two reasons. Some were too busy to see anything, unless specially brought to their notice. Others thought the whole matter already fully canvassed and settled. It is true there is nothing showing that the consent of Congress had been asked for this enterprise; certain that the subject had not been treated as an interstate affair, to say nothing of its being an international affair. The United States has always been slow to move; with its many sleeping rights, it has for many years been loth to exercise them. Not till 1888 did it begin to exercise positive legislation over its navigable waters in order to preserve them for all its citizens. Each river and harbour bill since then is found to have sections strengthening the hands of those who wish to keep the waterways open and in good order, for all classes of navigators. Not till 1890 had any prohibitive clauses been enacted into laws forbidding, for example, the destruction of channels by improper dumpings. Saw-mills went their own unchecked way every year, clogging up streams. Railways bridged all smaller streams, in the States, without interference from the United States. Many other features can be quoted. But it is sufficient to say that all that is now changed. The adopted policy is to defend, as well as improve, all water courses, now navigable, or probably navigable in the reasonably close future. Waterways are under the charge of the United States, and there is no likeli hood of their being abandoned for some time to come.

With this an established fact it is impossible to think that United States supervision shall not be extended to the Chicago drainage canal in due time. Under whatever law built, and for whatever purpose constructed, just so soon as it is shown that that canal affects, or becomes a part of the system of navigable waterways of the United States, some supervision or control of it must follow. When boats use it for harbour purposes; when its waters add to the Illinois River, or take from the lakes, they alter natural conditions and the matter rises for consideration under national authority.

The water levels of the great lakes are very delicate. Storms, barometric changes, rainfall, even tidal changes, are felt. Records show at Buffalo no less than 13 feet as a total possible change between the lowest and the highest gauge readings. Each lake is a basin. The water is constantly pouring in from not only one, but several inlets. The overflow, however, is now always out of the one inlet provided for that purpose; the second one, formerly at Chicago, has been plugged up.

As in our basins, when the water rises enough to take two, three or more of the small holes to carry it off, it is always to be noted that those holes are always carrying that surplus off; they do not wait until the water has time to pass from one end to the other. In the same channel the head alone governs the rate of overflow, and that head is measured by the gauge-reading at the outlet. The supply of water in the lake, the net supply, allowing for evaporation, is the sole cause of the outflow. That supply depends solely upon rainfall; but the lake, when it receives more than it has been receiving, must discharge more; when it has less, there is less to run out. If the outlet be dug down, or new ones made, the water runs off faster than it ran off before.

APPENDIX A—*Continued.*

The outflow is instantly affected by a changed inflow, provided there is enough such to increase or reduce the head. If we have a rainfall of 1 inch over the lake area (and such are not uncommon events) there is a head of 1 inch to run off. But if there are two outlets to run out of, instead of one, this inch must run off sooner than through the one. If the new outlet should reduce the levels of Lake Michigan and Huron about 6 inches, this effect will be produced in full in about two years; it is not then a question of many years, as some suppose.

We may feel very sure, therefore, that in this question two points are certain: 1. The drainage canal is not solely a state affair, but a national one. 2. The tapping the lakes must affect their levels. But it is said, first, that the changes in levels do not concern shippers, and then that, at most, the effects will be trifling.

If one watched carefully the course pursued by shippers one would see that, as a rule, each vessel carries all that it can take and get out of its port or into that it intends to reach. Vessel owners and managers are very shrewd, watchful men; they know what they can safely carry, allowing for storms and short detentions arising from passing causes; they average pretty well the practicable depths, and carry all the channels will stand. They are as conversant as are theorists about the effects of storms, but they keep good watch on ruling depths. Now, should it be certain that these average depths were reduced 3 inches, or 6 inches, they must load accordingly. And not only the large boats, but also the small ones using the small harbours that the large ones cannot go into. All must lose the 3 or 6 inches, as it may be; and not for one or more trips, but for all trips, and for all time; a diminution of capacity is not a single tax, but a continuous one. A vessel that when light draws 6 feet and loaded 12 feet must lose 3 inches out of 72, say 4 per cent in capacity, each loading; a vessel drawing 12 feet light and 20 feet loaded would lose somewhat over 3 per cent in capacity at each and every loading.

Should the loss of levels be 6 inches, instead of 3 inches, then these figures become doubled. Will the loss be six inches or will it be three inches? This is an important question, and we have only the Niagara River discharge observations from which to answer it. These cover a range of about 1·8 feet. There were scattering observations outside these limits, but the mass of results was secured between gauge readings, mean lake level, the highest, and 1·85 feet. The "smooth curve" as published enables us to note the fall of 0·53 feet on the gauge per 10,000 cubic feet per second for the first foot of fall, and 0·44 feet for the whole.

These observations, especially at the lower readings, are erratic, and indicate a need for more measurements, especially at these levels. This lower portion of the gauge should be studied and additional observations made, and the board is a unit in suggesting the importance of a series of gaugings of the St. Clair River at the present time for this purpose, and to furnish additional knowledge of the relation between gauge readings and discharge. The subject is of such general bearing upon the navigation of the lakes that it demands careful treatment and full data. The Niagara data do not show how much Lake Huron and Michigan would be lowered, even if 0·53 feet were the net loss to Lake Erie. The opinion expressed by Mr. Johnston that the effect on the two upper lakes would be some 15 per cent greater than upon Erie would seem to point to a probable loss of, say, 0·61 feet. This possible loss of 7 inches certainly is important enough to justify careful measurements of the discharge through the St. Clair. It is true that the law as it stands, and the intention of the trustees, contemplate the abstraction of only 300,000 cubic feet under present conditions; but after the canal is opened measurements will not be so instructive, and we must assume that ultimately the entire 600,000 cubic feet per minute will be drawn from Lake Michigan, as required by the state law.

The abstraction of 10,000 cubic feet of water per second from Lake Michigan will lower the levels of all the lakes of the system except Lake Superior, and reduce the navigable capacities of all harbours and shallows throughout the system to an extent that may be determined, if at all, by actual measurements only. Under the laws of the United States these changes in capacity cannot be made without federal authority, and

to enable the executive officers of the United States to act advisedly in the matter it is necessary, in the opinion of the board, not only that these measurements be taken, but that the money cost of restoring the navigable depths in channels and harbours be carefully estimated.

In this connection the board submits, without expression of opionion, an estimate prepared by Mr. Charles H. Keep, Secretary of the Lake Carriers' Association, of the commercial losses in carrying capacity of the lake fleet, should a reduction be made in lake levels of 1, 3 or 6 inches.

The board notices that the same peculiarity exhibited by the Niagara discharge curve is pointed out by Mr. Johnston as existing in the Morris, Ill., and South Branch Chicago River curves.

The board also notes Mr. Johnston's conclusions that:

Applying the reasoning to the St. Clair and Detroit Rivers, the value of the Q^1—Q may be taken from the diagrams illustrating the tables before described, the only uncertainty being as to the value of a. Suppose a to be a unity, and the mean depth 20 feet. Then Q^1—Q will equal something greater than 20,000 cubic feet per second.

This practically corresponds with the deductions made from the Niagara River observations.

So many uncertainties arise in the application of hydraulic formulas that the only way to ascertain the approximate discharge of these streams is to measure them for periods long enough to eliminate accidental fluctuations and to cover all stages.

While the navigable capacity of all harbours and channels on the Great Lakes below St. Mary's Falls will be injuriously affected by a diminution in depth, the navigability of the inner harbour of Chicago will be diminished also by the introduction of a current therein, which, in the present condition of the river, even with the minimum flow of 5,000 cubic feet per second, or 300,000 cubic feet per minute, is entirely inadmissible. The estimates of the effect of the drainage canal upon this harbour should also consider this element.

The Board of Trustees have not yet determined upon a plan of treatment of this navigable channel, and their plans may be such as may improve, impair or destroy its utility as a navigable river.

All of which is respectfully submitted.

O. M. POE,
Col. Corps of Engineers, Bvt. Brig.-Gen., U.S.A.

E. H. RUFFNER,
Major of Engineers, U.S.A.

W. L. MARSHALL,
Major, Corps of Engineers.

APPENDIX B.

(*Copy.*)

OFFICE OF THE CHIEF OF ENGINEERS, UNITED STATES ARMY,
WASHINGTON, D.C., February 7, 1896.

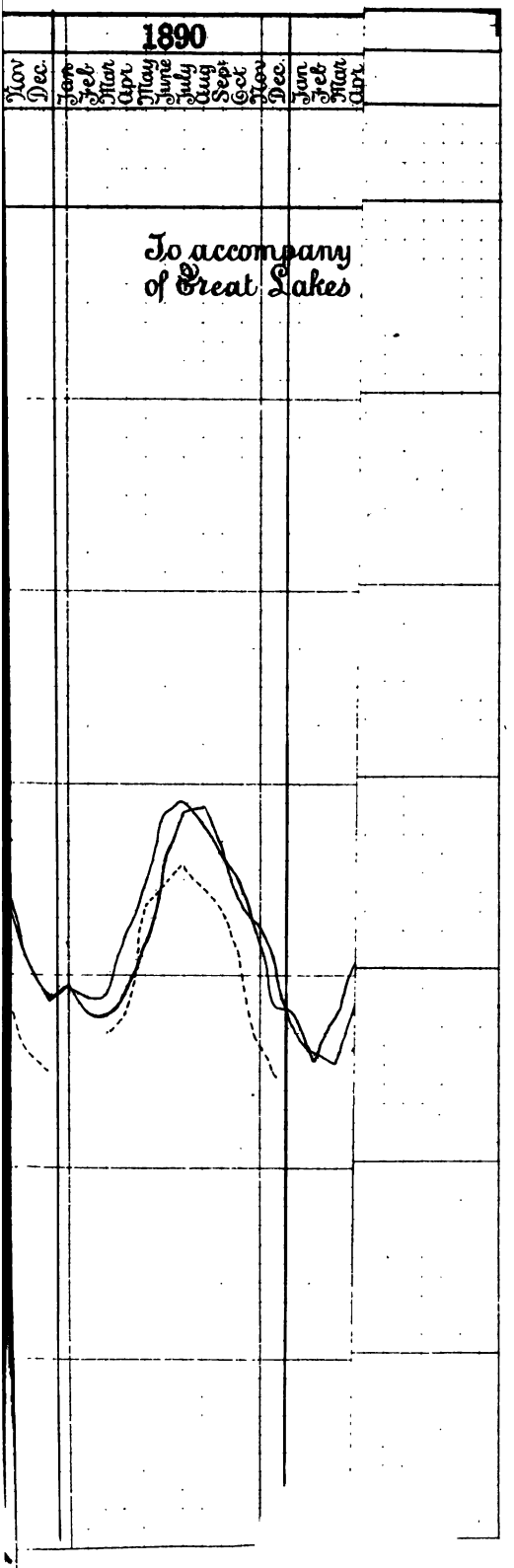
Mr. J. L. P. O'HANLY, C.E.,
Care Department of Marine and Fisheries,
Ottawa, Canada.

SIR,—Your letter of the 3rd instant is received, and in reply to your inquiry relative to the gaugings of the St. Clair River, suggested by the Board of Engineers appointed to report on the probable effect of the Chicago Drainage Canal on the levels of the Great Lakes, I have to say that the work of gauging the St. Clair River has not as yet been undertaken.

By direction of the Chief of Engineers.

Very respectfully, your obedient servant,

A. MACKENZIE,
Lieut.-Col., Corps of Engineers.


1890
Nov
Dec.
Jan
Feb
Mar
Apr
May
June
July
Aug
Sept
Oct
Nov
Dec.
Jan.
Feb
Mar
Apr.
To accompany
of Great Lakes

Nov
Dec
Jan
Feb
Mar
Apr
May
ompany rep
t Lakes ar

hall on
River.

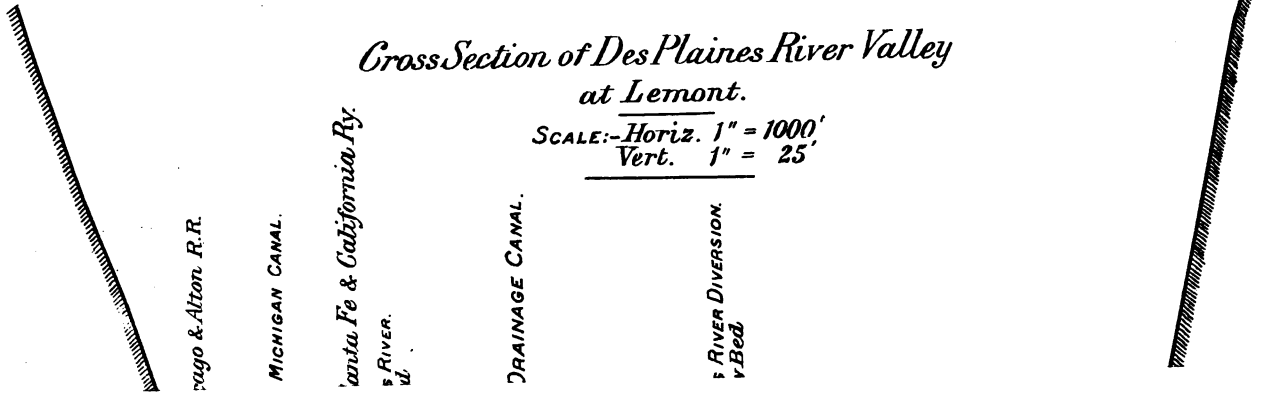
Drainage Canal on levels of Great Lakes and connecting Rivers" by order of

HONORABLE JOHN COSTIGAN,
Minister of Marine and Fisheries,
Canada.

SEP 3 1896

Cross Section of Des Plaines River Valley at Lemont.

SCALE:- Horiz. 1" = 1000'
Vert. 1" = 25'

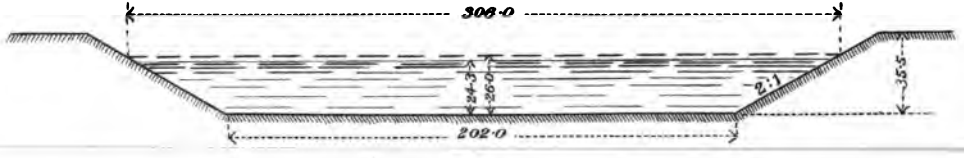


To accompany report on "Effect of Chicago Drainage Canal on levels of Great Lakes and connecting Rivers" by order of

HONORABLE JOHN COSTIGAN,
Minister of Marine and Fisheries, Canada.

PLATE IX.

SANITARY CANAL – CHICAGO.



Ottawa, Ont., Canada, April, 1896.

J. L. P. O'HANLY, C.E.

CABOT SCIENCE LIBRARY
CABOT
CANCELLED
APR 26 2001
MAY 18 2001
BOOK DUE

3 2044 056 185 523

3 2044 056 185 523